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Interactive comment on "Topology of sustainable management in dynamical Earth system models with desirable states" *by* J. Heitzig and T. Kittel

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Final Response

Response to general comments

We thank the reviewers for their overall positive comments on the content of our manuscript and the detailed suggestions how the presentation might be improved. Our original idea was to introduce the needed mathematics not en bloc but spread-out throughout the paper, separated by examples that motivate and illustrate each of the pieces, somewhat like in typical lecture notes. From the reviews we realize that this strategy may not work as hoped and that a more strict division between verbal descriptions of concepts, their mathematical formalization, and illustrating examples may be

better. For this reason, we plan to restructure the manuscript upon revision as follows:

Before introducing the mathematical notation in what is now 1.1, we add a section with a concise verbal exposition of all our concepts, accompanied by what is now the lower part of figure 3 (the summary of our concepts in the form of a decision tree) and by an improved version of the upper part of figure 3 (as a motivation of our choice of metaphorical terms from nautics). We then continue with a more formal and detailed treatment of these concepts, choosing the lowest adequate level of mathematical complexity (and reserving the full mathematical treatment for the appendix), accompanied by a series of simple figures in the general spirit of what is now figure 1 but each one focussing on just a few of the introduced state space regions.

Only after this, we will present the illustrative examples, stating each example's purpose more clearly than before, and moving what is now example 3 into the appendix. Still, for a number of reasons we will not attempt at choosing different, more realistic examples from earth system science but stick to the given set of conceptual low-dimensional models. First, as this is mainly a theoretical paper introducing new concepts and tools, the purpose of the examples is only to illustrate how the characterized state space regions and dilemmas may occur in various fields related to the earth system. An actual application of the concepts to a realistic model of some subsystem of the earth system is way beyond both the scope of the present paper and our abilities and should be the natural subject of future work. Second, in order to clearly illustrate the concepts graphically, the examples have to be two-dimensional. Third, in order to avoid unnecessary numerical complications and artefacts, and to give evidence for the fact that the discussed dilemmas may easily occur already in quite simple systems and are thus probably quite pervasive in more complex systems, their dynamics has to be as simple as possible. As some examples will appear unrealistic to experts in the respective field, in particular what is now examples 5 and 9, we will take care to discuss them in a little more detail.

The first referee judged that "the relevance of the framework to Earth system dynamics, and the relevance of the results for earth system governance, is also not sufficiently elaborated." To this we can only say that we believe that we did elaborate quite much on the relevance for governance in the introduction, the individual examples, and the discussion, but will check again that our statements about this come through even more clearly, in particular our bottom line message that "before performing some form of quantitative optimization, the sustainable management of the Earth system may require decisions of a more discrete type that come in the form of several dilemmata, e.g., choosing between eventual safety and uninterrupted desirability, or between uninterrupted safety and increasing flexibility." Regarding the latter, the second referee helpfully suggested to either make clearer "how tradeoffs between these criteria play out" or that "the limits of the paper could be made more clear from the beginning to avoid expectations that cannot be met." In view of the much more complex nature of the real-world earth system, we don't feel experienced enough to make reliable statements about the actual tradeoffs, which we believe may be a whole research agenda that may follow from our theoretical suggestions, so we will rather describe the extent and limits of our work more clearly in the introduction and suggest more specific next steps in the outlook.

Response to specific comments and questions from the "technical corrections" section

Page 442, line 15: A reviewer asked, why is the reachability between two sets limited to both being "arbitrarily close"? Wouldn't reachability also make sense for any pair of sets, whatever their distance? This seems to be a misunderstanding. We define a notion of stable reachability by giving certain mathematical conditions under which we say that "a set Y is stably reachable from a state x through some other set A". Y and x do not have to be arbitrarily close (but Y and A do have to, since otherwise one could not reach Y without leaving A). For technical reasons, we formulate the condition used in the definition of "stable reachability" in terms of another, auxiliary notion, that of "forecourt", which allows us to elegantly define "stable reachability" like this: A set of states Y is stably reachable from a state x iff (meaning "if and only if") x is

in some forecourt C of Y. Now the words "arbitrarily close" which apparently confused the referee only occur in the definition of "forecourt": A forecourt of a set Y is another set C from whose every point one can approach Y arbitrarily closely without leaving C by suitable management. As a consequence, the forecourt C must of course be arbitrarily close to Y in the sense that it intersects every topological neighbourhood (in particular, every ε-neighbourhood) of Y. We could alternatively have avoided the notion of arbitrarily close approach and instead have demanded the more restrictive condition that one can navigate properly *into* Y from every point in C without leaving C. In that case, the forecourt C would even have had to intersect the target set Y. But then all asymptotically stable fixed points which one cannot ever reach exactly in finite time but can only converge to in infinite time (and hence can only approach arbitrarily closely in finite time) would not have been counted as "stably reachable". However, such asymptotically stable fixed points occur in many models, and in practice (and in view of typically unavoidable small perturbations) the possibility of an arbitrarily close approach is all that will ever be relevant for real-world management. For this reason we chose the more complex definition involving the condition of arbitrary close approach instead of the simpler but more restrictive one.

Page 446: Example 3 was only given for completeness as a concise summarizing example for the more mathematically inclined reader whose main value is that it shows that all the introduced regions can occur simultaneously even in a simple low-dimensional example with a high degree of symmetry. It was however not considered important enough to spent space for an accompanying figure. We decided to move this example into the appendix in the revised version of the manuscript, and will consider adding a figure anyway.

Page 449: Example 5 on alternative plant types and multistability is meant to show with a conceptual model how a lake dilemma may occur in a simple multistable system. Although it is known that many plants modify the soil in ways that benefit their own growth, e.g. via influencing microbial communities and biogeochemical cycling (e.g., Kourtev et al., Ecology 2002; Read et al., New Phytologist 2003) and empirical evidence exists that this has effects on interspecies plant competition (e.g., Poon, Master's Thesis University of Guelph, 2011), we know of no formal model that would allow to study the resulting feedbacks between two plants and is simple enough for the purpose of illustrating our theory in an adequate amount of space. The best existing candidate models seem to be the four-dimensional model of a two-species plant-soil-feedback by Bever et al., New Phytologist, 2003 (see also Kulmatiski et al., J. Ecol. 2011) and the spatially resolved model of an invading plant by Levine et al., Ecology 2006, which however does not model other species explicitly. For this reason, we chose to design a conceptual model of two fictitious plant types each of which grows according to the well-established logistic growth dynamics leading to an initially exponential growth that is dampened by intraspecies competition. In order to keep the state space dimension at only two dimensions so that state space diagrams can be plotted, we refrained from modelling the soil characteristics via dynamic variables as in the other models, and instead represented the soil modification effect by simply assuming that the two species' undampened growth rates are proportional to some carrying capacities K_1, K_2 that the current soil composition implies for the two species, and that K_1, K_2 depend directly on the existing two populations x_1, x_2 in some simple way. In order to study the effect of soil modification alone, we did not include other interspecies interactions such as direct interspecies competition for resources. Levine et al., Ecology 2006, also assume dampened growth with a basic rate that depends on the existing population, but they only focus on a single species and assume a fixed carrying capacity, which we find somewhat implausible in view of the empirical evidence presented in Poon, Master's Thesis University of Guelph, 2011. Because we wanted to produce a conceptual model that illustrates the topological landscape in a multistable system, we needed to make sure the actual functional form we chose for K_1, K_2 produces a multistable system. This was achieved by assuming that the effect of the two populations x_1, x_2 on the two carrying capacities K_1, K_2 is nonlinear in the sense that the marginal soil improvement by plants of the same species is declining with higher populations while the marginal

effect of plants of the other species is increasing with their population. We are not claiming that this is so in real-world plant-soil-feedback systems, but believe that the alternative assumption of a linear relationship seems unlikely. We then chose a very simple formula for K_1, K_2 that has these properties. In the revised paper, we will try to make these choices more explicit and add the mentioned references for comparison.

Page 455-461: We will follow the suggestion to "clarify better in words what some of the term mean (transitional, rapid, channel, fairway, finer partition)."

Page 464, line 5: "What situations in the Earth system might correspond to lakes, glades, etc.?" This is a hard question probably requiring specialists from several fields of earth system science, so we would only be able to speculate here. We will consider doing so in a transparent way, but will probably rather suggest more specific next steps for research in this direction.

Technical corrections not yet discussed above

Before submitting the revised version, we will of course perform spell-checks, and we are also happy to change the plural of the greek word "dilemma" from its greek plural "dilemmata" to the English "dilemmas" as suggested if that is considered important. The mentioned German word "beim" is part of the cited author's surname and thus correct. Regarding figure sizes, we had designed them originally already to work well with the eventual ESD two-column layout instead of with the ESDD landscape layout, which explains the somewhat suboptimal appearance in the discussion paper, for which we apologize.

One referee pointed out a possible misunderstanding of equations of the type $S = (a, b)$ in which a set S is equated to an open interval (a, b) of real numbers since the notation (a, b) could as well refer to a point in 2D space. We will take care in the revision to make this unambiguous by both adding the words "open interval" and maybe using the alternative notation $]a, b[$; the use of the equal sign to denote equality between sets (as between any other mathematical objects) however is so standard and needed so often in the paper that we can hardly imagine how to avoid it.

Page 444, line 23: We will reverse this sequence so that the ordering of the arrows matches their definition better (from left to right).

Page 450, last two paragraphs: We thank the anonymous reviewer for the comments regarding this discussion of the two stable fixed points, the shelter, and the lake. We will reformulate them in a more accessible way to avoid the pointed-out ambiguities.

Page 451, line 25: We introduced $g(r)$ as the upper solution to the equation $F(g(r)) - 1 = 0$, but will elaborate on this a little more in the revision.

Page 453, line 4: Following the reviewer's suggestion, the names and a short explanation for all the individual parameters will be included. With the normal serif font ESD uses for their final papers, the greek lowercase gamma and the letter y should be easily distinguishable, so we decided not to change the symbol.

Page 453 line 10: As it was correctly pointed out, values of $y < 0$ are unphysical, so the correct bound on y should read $\max(0, -(\ell+\delta)/\phi\gamma_0)$ instead of just $-(\ell+\delta)/\phi\gamma_0$, since the latter may be negative depending on the relative sizes of the negative quantity ℓ and the positive δ. This will be corrected.

Page 457: As mentioned, Example 9 was designed by us only to illustrate the relationship of reachability and bifurcations. It is not meant to represent any real-world example. However, it contains an individual and a paired saddle-node bifurcation similar to those that may occur in bistable Earth system components such as the Atlantic Meridional Overturning Circulation (AMOC) or other tipping elements (Schellnhuber, 2009), as mentioned.

Page 460 (!), line 18: We will use a different letter than D for docks to avoid the pointed-out collision in notation.

Page 479, figure 1: We will either remove this figure completely or replace it by a sequence of similar but simpler and more illustrative diagrams used to illustrate the

individual state space regions at the end of the mathematical section that follows the verbal exposition of the regions (see discussion above).

Page 481, figure 3: We will move both parts of the figure to the end of the verbal exposition of our main concepts (see discussion above). Upper part: This figure was meant to depict all introduced regions in an illustration of a state space that roughly resembles a flow from an "upstream" via a "downstream" region into an "abyss" etc. Obviously, it was much too abstract to really make this intended analogy apparent. Since we still believe that such an illustration, if carried out properly, can help motivating our choice of metaphorical terms, so we will aim at replacing it by a much more explicit drawn illustration of a water system involving a river running from a mountainous region into an ocean with a trench.

Page 484, figure 6: In the revision, we will make even clearer than before that the four subplots are for four different archetypical systems which represent archetypical situations of bifurcations. We will also add more labels and vertical arrows to indicate the default dynamics towards the stable fixed points.

Page 485, figure 7: We thank the anonymous reviewer for the comment concerning γ_0 and γ_1. These refer to the values of the parameter γ in the default, unmanaged dynamics and the managed dynamics, respectively. To clarify, we will rename them and add a short descritpion. The other parameters refer directly to their mentioning on page 453, line 4.

Page 487, figure 9: By "extreme admissible management trajectories", we mean in this context those admissible management trajectories that correspond to the largest-possible leftward or rightward motion from their initial point. We will seek a less ambiguous phrasing in the revision.

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