

## Author's final comments

Dear referees, editors,

Thanks for paying attention, careful revision of the manuscript and for the comments to the manuscript. Our responses follow below.

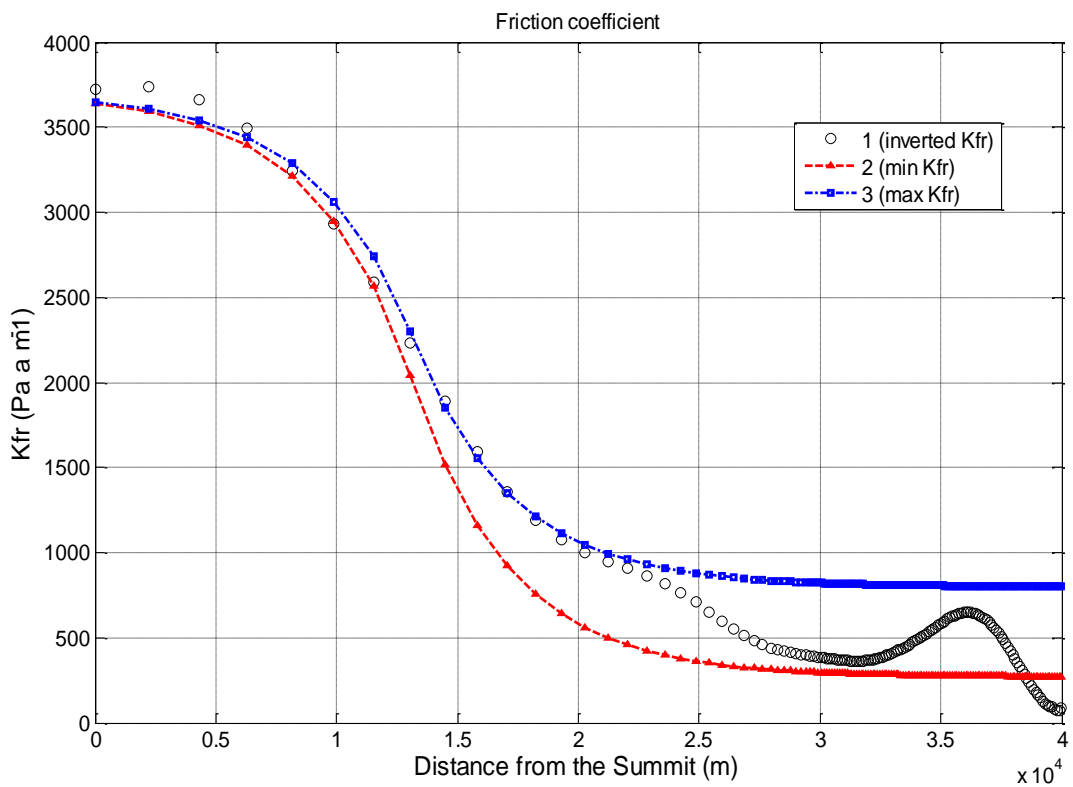
### **T. Dunse:** “1. Use of constant basal friction coefficients

*...Basal friction coefficients calculated from 1995 velocity fields therefore represent a snapshot in time and are unlikely to be representative over long time scales of 500 years. The authors should investigate the sensitivity of their model results to significant variations in friction coefficients as they would be obtained based on significantly different velocity fields. The associated uncertainties should be discussed...*”

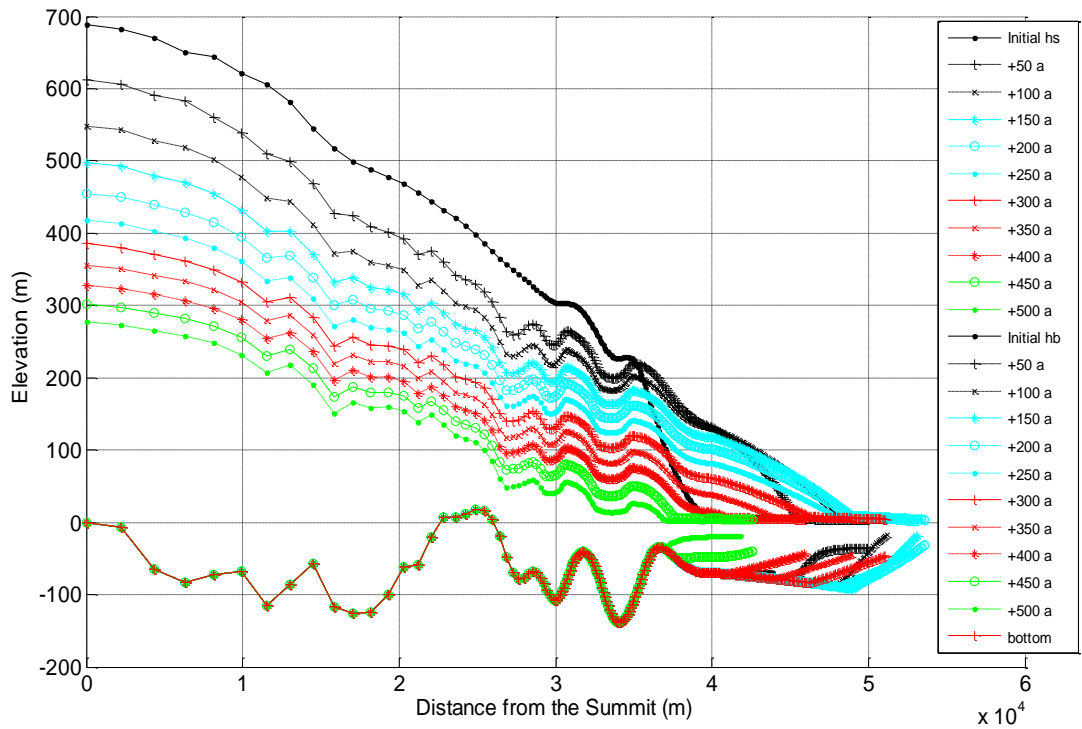
...We suggest to consider two ancillary curves that can be approximately defined as the lower boundary and the upper boundary, respectively, for the inverted friction coefficient. These curves are shown in Fig. 1A (curve 2 and curve 3, respectively). The modeled successive C–C' cross-section geometries are shown in Fig. 2A and in Fig. 3A, respectively. For the lower and for the upper curves (Fig. 1A) the grounding line histories differ from the history obtained for the inverted friction coefficient (Fig. 4A). Thus, the suggested variations in the friction coefficient provide the dissemination of the grounding line histories. However, at long time scales of 500 years the grounding line histories (Fig. 4A) and ice thickness distributions (Fig. 2A, Fig. 3A) reveal ice mass decline in all suggested experiments (without achievement of a steady-state distributions).

In addition to this efforts, the alteration in the friction law, i.e. the applying of the non-linear Weertman type friction law instead of the linear one, - yields to the qualitatively same results (see Fig. 5A, Fig. 6A).

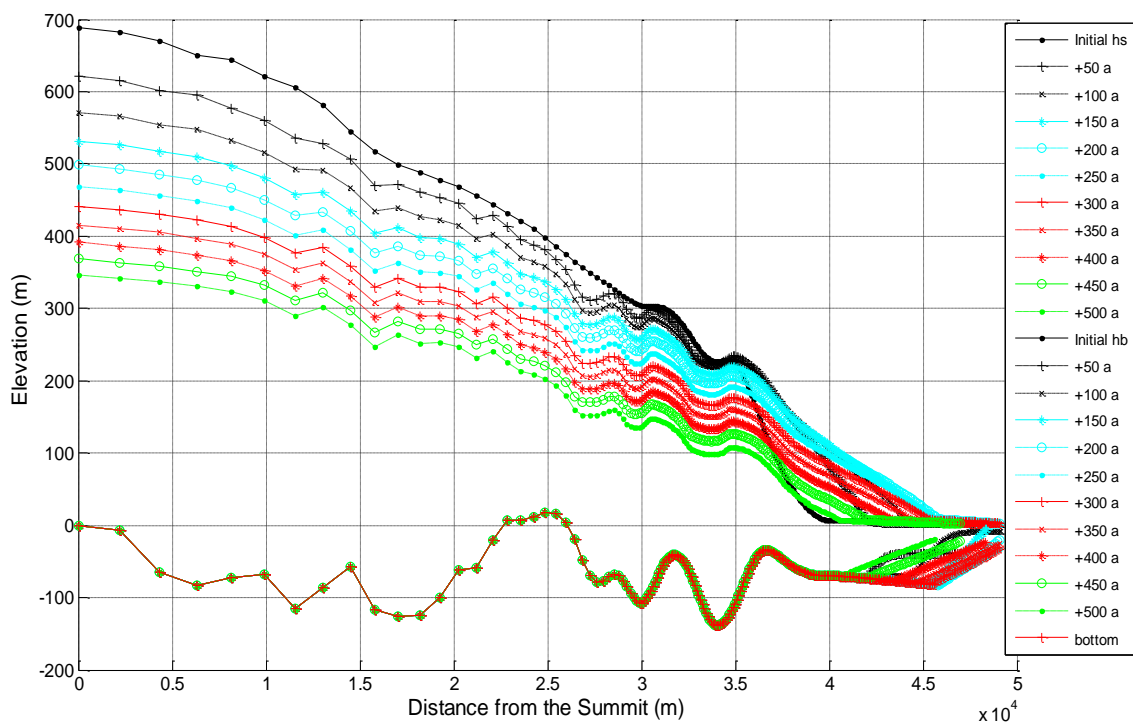
Thus, essentially, these complementary experiments justify the conclusion about the continuous ice decline along the flow lines over the time scale of 500 years. On the other hand, this corollary implies that the surface mass balance, which provides the essential impact on the ice retreat/advance, is unchanged during the temporal period of 500 years. Of course, we don't know how will the mass balance change in the future? If it will increase, then the ice decline can be changed to the ice mass growing. However, the experiments with the shifted mass balance show that for the C-C' ice stream the steady state ice thickness distribution is achieved at the surface mass balance shifted up at 0.7 m/a.



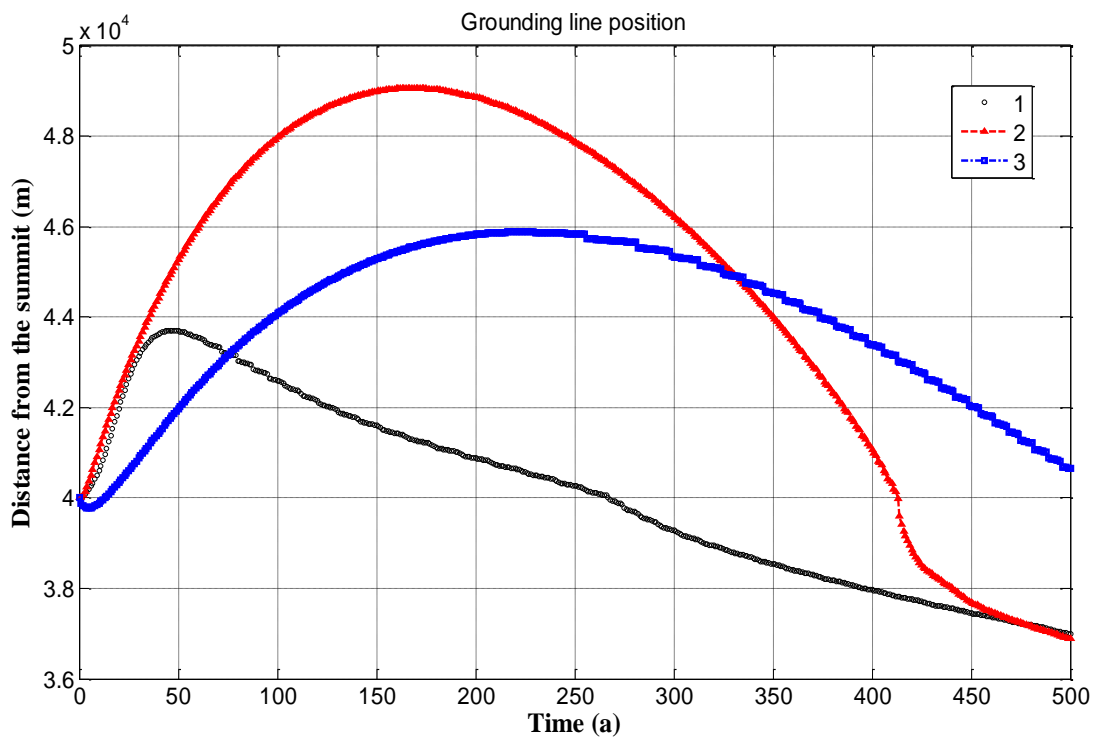
**Fig. 1A. 1** - the friction coefficient inverted along C-C' flow line. **2** and **3** – approximations of the friction coefficient.



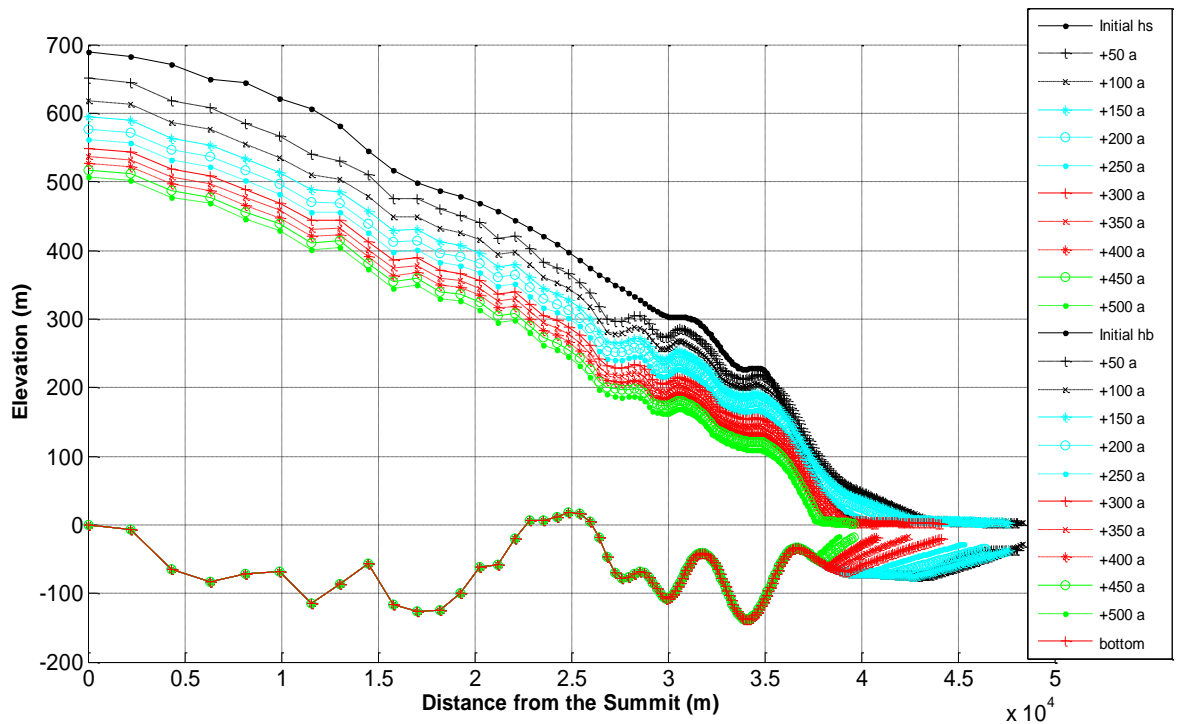
**Fig. 2A.** The modeled successive C-C' cross-section geometries separated by 50-year intervals from the present to the future 500 years later. The friction coefficient distribution is defined by the curve 2 in Fig. 1A.



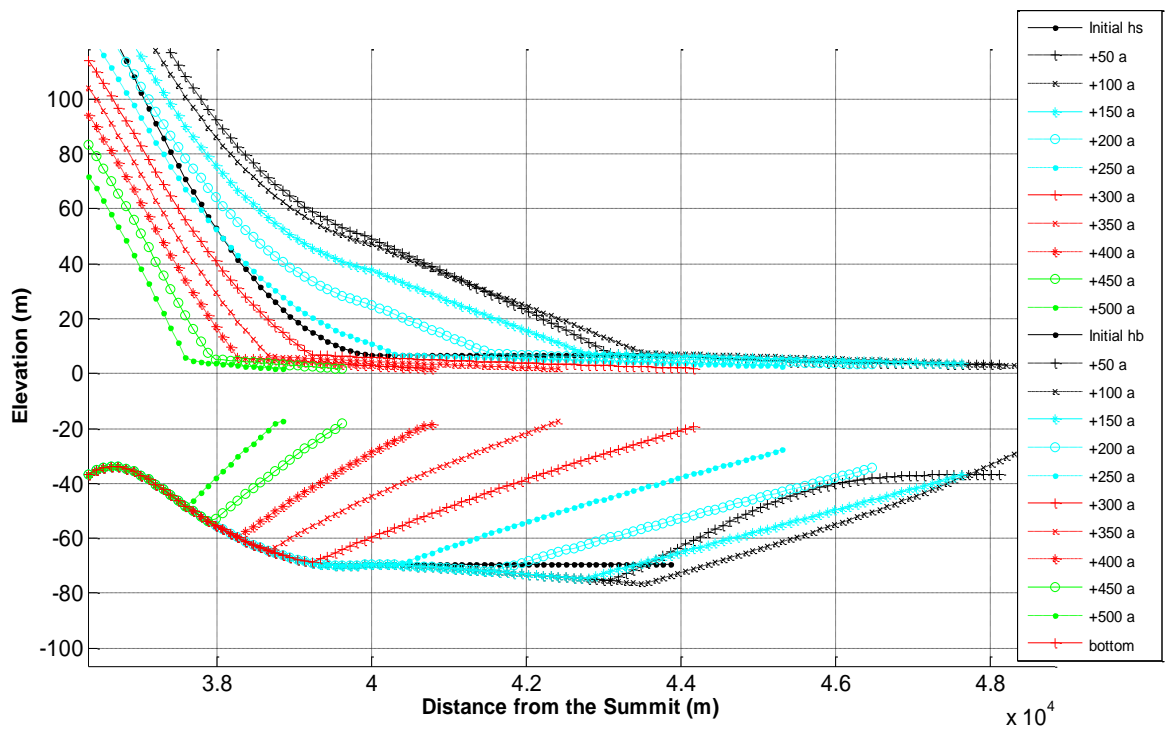
**Fig. 3A.** The modeled successive C-C' cross-section geometries separated by 50-year intervals from the present to the future 500 years later. The friction coefficient distribution is defined by the curve 3 in Fig. 1A.



**Fig. 4A.** The modeled grounding line history (1) for the inverted friction coefficient (2) for the friction coefficient distribution defined by the curve 2 in Fig. 1A (3) for the friction coefficient distribution defined by the curve 3 in Fig. 1A.

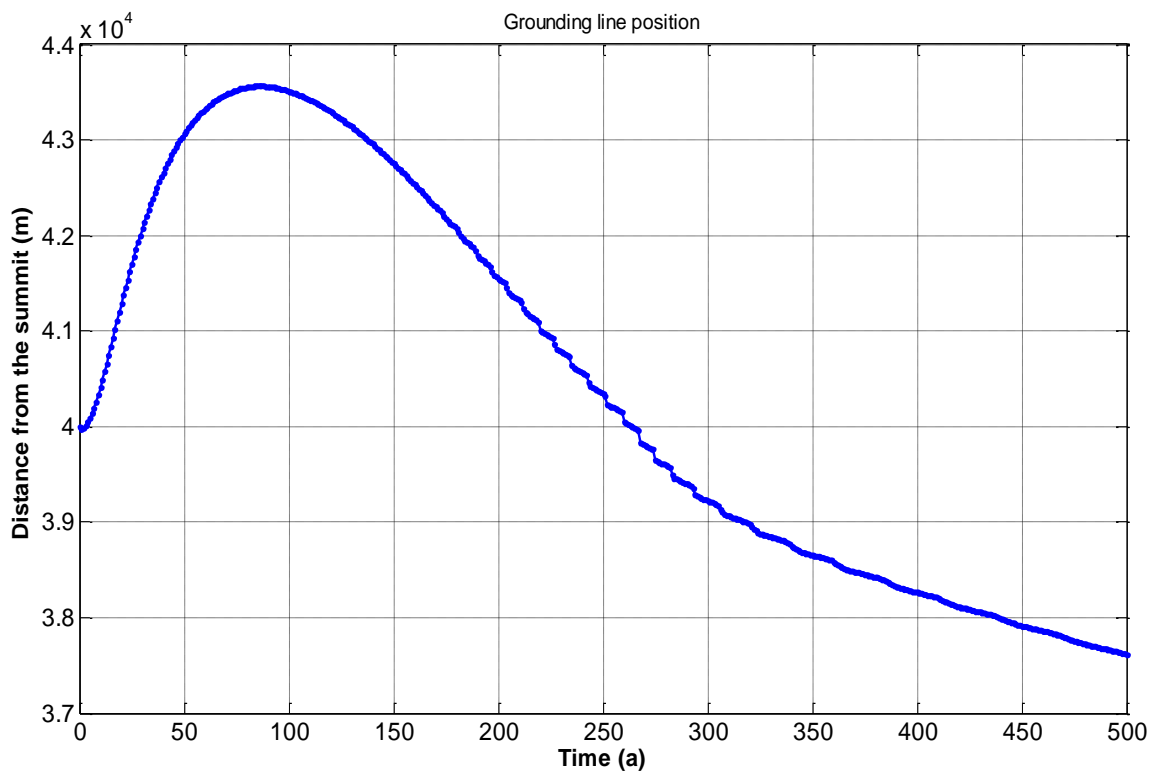


**Fig. 5A, a**



**Fig. 5A, b**

**Fig. 5A. (a)** The modeled successive C-C' cross-section geometries separated by 50-year intervals from the present to the future 500 years later. **(b)** A magnified section of panel (a), showing the evolution of C-C' ice shelf. The modeled geometries obtained for *non-linear Weertman type friction law* (the friction coefficient was inverted in (Konvalov, 2012)).



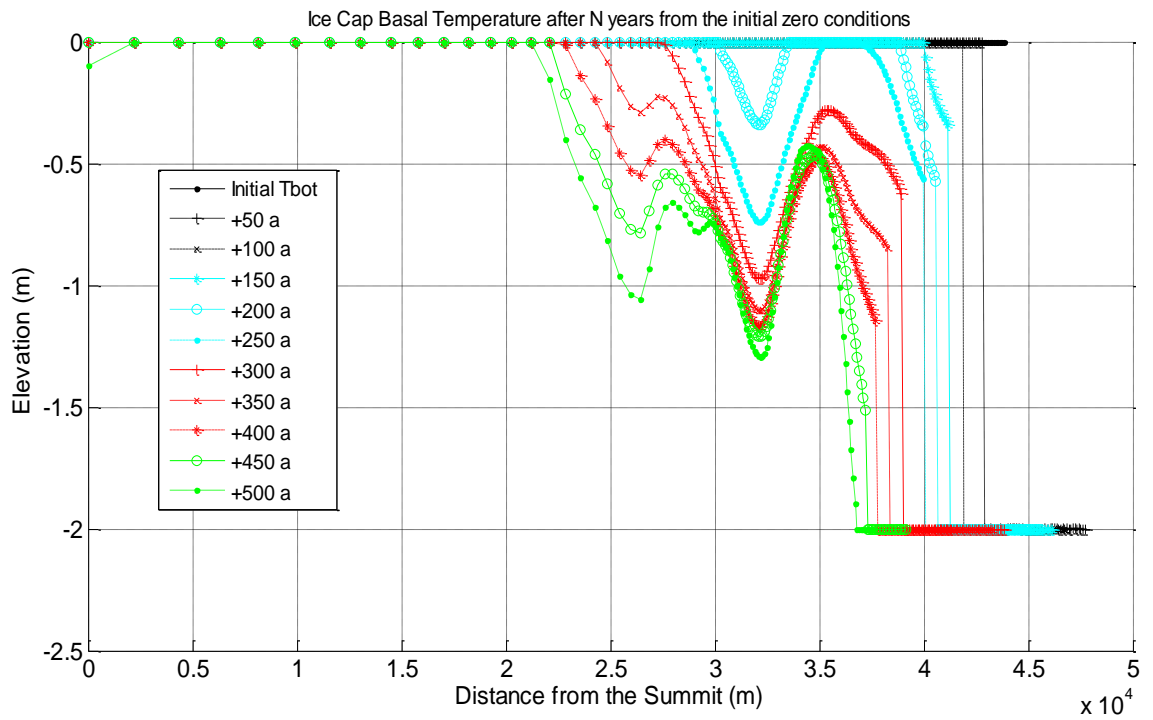
**Fig. 6A.** The modeled grounding line history for C-C' ice stream and for *non-linear Weertman type friction law*.

**T. Dunse:** “2 Initial and simulated ice temperature and effect on ice dynamics

*The authors admit that latent heat release by meltwater refreezing within the snow and firn is not considered. However, in an earlier study, Nagornov et al., 2005 (Ann.Glac.) point out the importance of subsurface melting for the temperature profile. Firn warming may have been negligible in the Little ice age, and consequently, present ice temperatures in the lower part of the glacier may not be affected by it. However, firn warming is important today, and will also affect the basal thermal regime over a long timescale of 500 years. The temperature fields displayed in fig.5 do therefore not show the expected temperature distribution with warmer near-surface-ice temperatures in the accumulation area (above 400m elevation; where firn warming operates) and colder near-surface-ice temperatures in the ablation area, despite of warmer surface air temperatures...”*

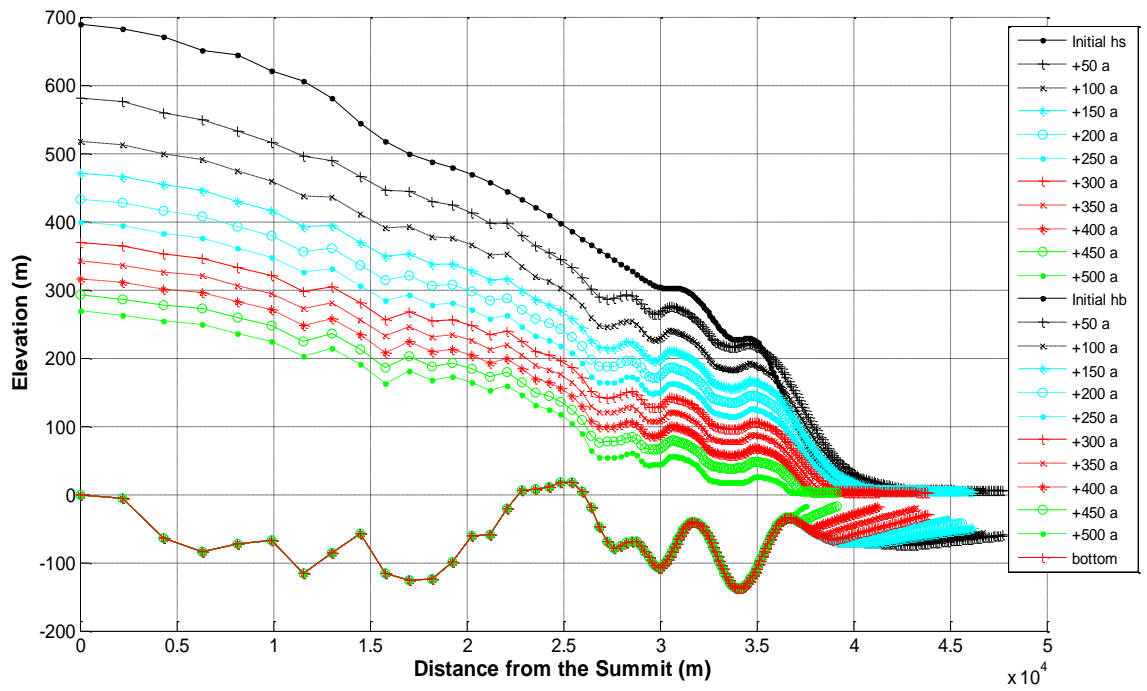
In the manuscript 2005 the 1D inverse problem for ice surface temperature is considered. To define the parameters of the heat source, which provides the heating due to melt water recrystallization (Paterson and Clarke, 1978), we used different types of the data: the temperature profile, the content of the infiltration ice in the annual layers,  $\delta^{18}O$  past history. All these data were obtained from the borehole measurements beside the summit of the Academy of Sciences Ice Cap. Thus, the data provide the heat source parameters, including temporal variability, in the past (not in the future) and only beside the summit (not along the whole flow line).

However, we agree that the firn warming is important and we have written about this in the manuscript. To evaluate the possible heating of the ice due to unaccounted sources we implemented the experiments with different temperatures. For instance, the zero initial ice temperature provides close to zero basal temperatures over a long timescale of 500 years (Fig. 7A). The modeled successive C–C’ cross-section geometries and the grounding line history are shown in Fig. 8A and in Fig. 9A, respectively. Comparison the results obtained in the experiments with different ice temperatures (Fig.9A) and with different friction coefficients (Fig. 4A) reveals more significant impact of the friction coefficient variation. Thus, in the context of these studies the inverse problem for the friction coefficient (the method of the inversion) seems more essential for the forecasts.

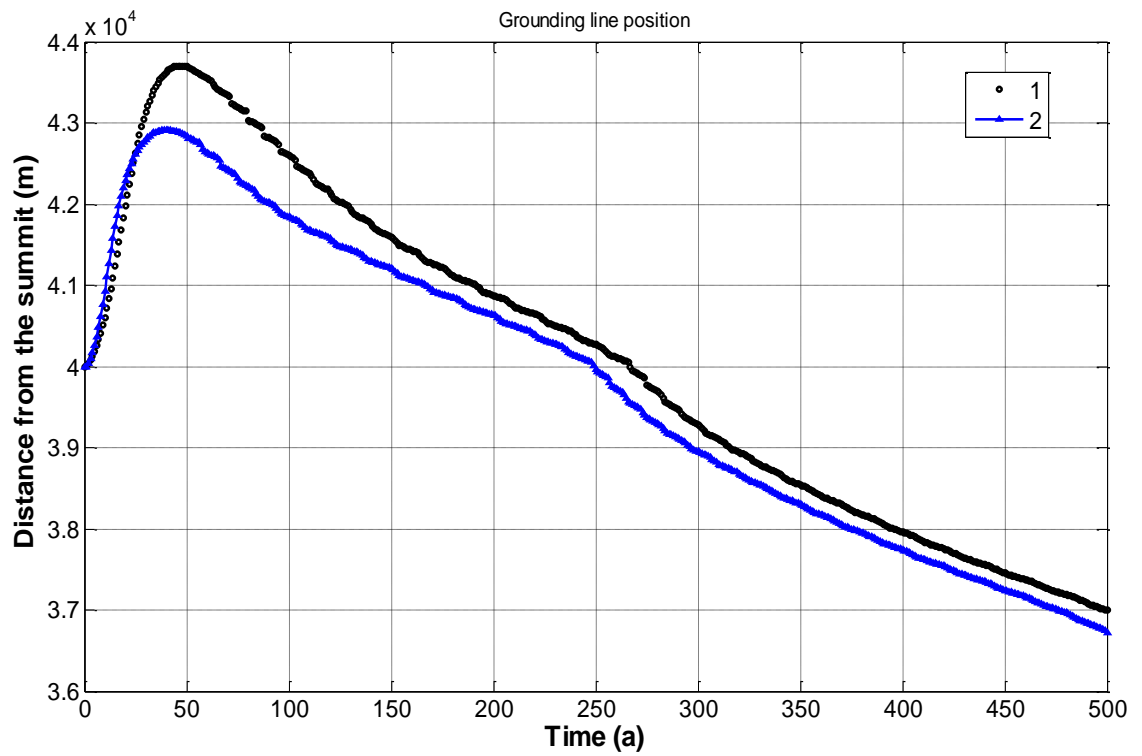


**Fig. 7A.** The modeled successive C-C' basal temperatures separated by 50-year intervals from the present to the future 500 years later.





**Fig. 8A.** The modeled successive C-C' cross-section geometries separated by 50-year intervals from the present to the future 500 years later. *The case of zero initial ice temperature.*



**Fig. 9A.** The modeled grounding line history (1) for non-zero initial temperature (Fig. 5,b in the manuscript) (2) for zero initial ice temperature.

**T. Dunse:** “The authors should consider the results from Moholdt et al., 2012 and discuss the implications for their study. The authors acknowledge Dowdeswell, et al., 2002 for data and reproduction of figures. However, it is not clear to me if they have actually acquired permission of reprint from the author and publisher.”

We agree that it is interesting to compare the results from Moholdt et al., 2012 with the results of the modelling. We going to include the corresponding results obtained by the modelling into the manuscript. Thanks.

**T. Dunse:** “The model description is insufficient. The authors do for example not mention that their employed model is a higher order model – or is it full Stokes/SIA? Additional components, such as the calving model are barely described at all.”

The sufficient part of the model description was included into the manuscript 2012 (Konovalov, 2012). The employed model can be defined as 2D (HV, i.e. one horizontal and one vertical directions are considered in the model) full Stokes thermo-coupled flow line model.

Authors are grateful to the referee for the text correction.

**Referee #2:** “The model as stated is incomplete and some of the symbols that are used are not defined. The worst example of this is that the authors invert for a friction coefficient that is not defined anywhere in the paper, nor is there an equation that describes how the basal friction coefficient enters into the ice flow model. Instead, the reader is referred to Konovalov (2012), but even from this paper I could not extract the exact form of the friction law used in the paper under consideration, nor the calving law, or the grounding line boundary conditions. On page 2216, line 14 it is stated that ‘a linear (viscous) friction law’ is used, which presumably is supposed to mean that a friction law of the form

$$\sigma_{xz,b} = \gamma u_b$$

is used, with  $\gamma$  the friction parameter,  $u$  the basal value of the along-flow velocity and  $\sigma_{xz,b}$  the basal shear stress?

It is essential to state the full forward model in a complete but succinct manner in order to fully review the scientific merit of the manuscript.”

The form of the friction law, used in the paper, is

$$\tau_i \sigma_{ik} n_k = K_{fr} |\vec{v}_b|^{m-1} (\vec{v}_b)_i \tau_i,$$

where  $K_{fr}$  is friction coefficient,  $m$  is the exponent in the friction law,  $m = 1$  gives linear friction law and  $m > 1$  corresponds to non-linear friction law;  $\vec{v}_b$  is the base ice flow velocity;  $\vec{\tau}$  and  $\vec{n}$  are tangential and normal to the bed vectors, respectively (Equations (2) and (4) in (Konovalov, 2012)).

**Referee #2:** “The ice flow model is a 2-dimensional flow line model, which is not really appropriate for a small outlet glacier, in which all stress components are potentially important. Zhang et al. (2015) find that these kinds of higher order flow line models show high discrepancies to full Stokes models when sliding is important and that these differences increase

*over long time integrations. This casts doubts on the applicability of the model used here, which introduces the additional complexity of grounding line migration.”*

2D flow line model implies that there is the surface inside the ice stream/glacier along the ice flow. On this surface the derivatives  $\left(\frac{\partial v}{\partial y}, \frac{\partial \sigma_{xy}}{\partial y}, \frac{\partial \sigma_{yy}}{\partial y}, \frac{\partial \sigma_{zy}}{\partial y}\right)$  in transverse direction (y) are approximately equal to zero. Thus, the 3D equations of the full Stokes ice flow model reduce to the 2D equations that lead to the flow line model. From the mathematical point of view, the existence of the surface and of the corresponding flow line doesn't depend on the size of the ice stream. Thus, the 2D flowline model is the approximation and, therefore, a distinction should appear in the histories obtained by the 2D model and by the 3D model, respectively. Nevertheless, we anticipate that the conclusion about the continuous ice decline along the flow lines will be unchanged.

**Referee #2:** *“Grounding line dynamics: the retreat of the outlet glaciers’ (I prefer this terminology over ice streams, which are generally associated with ice sheets) grounding lines by several kilometres is an important finding of the authors, but no details of how this grounding line motion is modelled are presented. In recent years publications have emerged which considered the performance of different models and implementations for modelling grounding line motion (e.g., Pattyn et al., 2012), but these are insufficiently referenced and it is not clear whether any of these findings are taken into account.”*

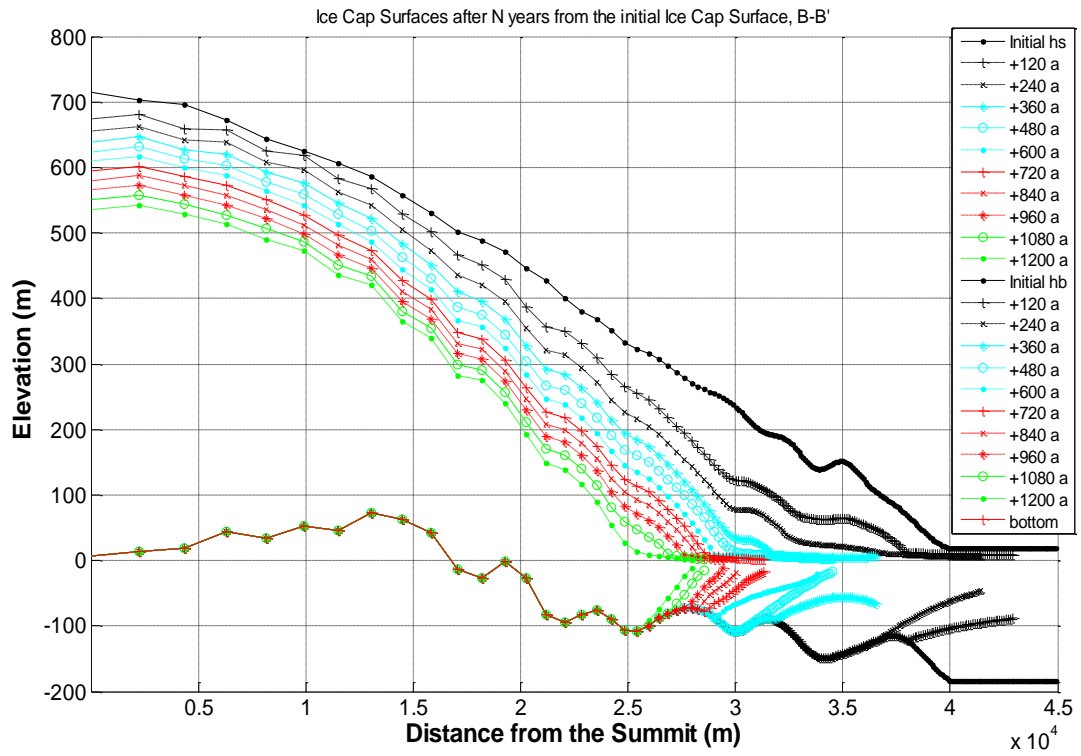
Grounding line position in the model is defined by the same method like described in (Pattyn et al., 2012).

**Referee #2:** *“The model does not take any inflow of mass from the regions surrounding the ice streams into account, which makes it unsurprising that the authors find that their equilibrium configuration is much smaller than their current state: most ice streams drain an area larger than just their surface area, consider for instance Jacobshavn Isbræ (Joughin et al., 2008). If inflow of ice from these regions is not taken into account, one would naturally end up with an overly negative mass balance. A full understanding of the evolution of the ice cap and its outlet glaciers would hence require modelling the entire ice cap.”*

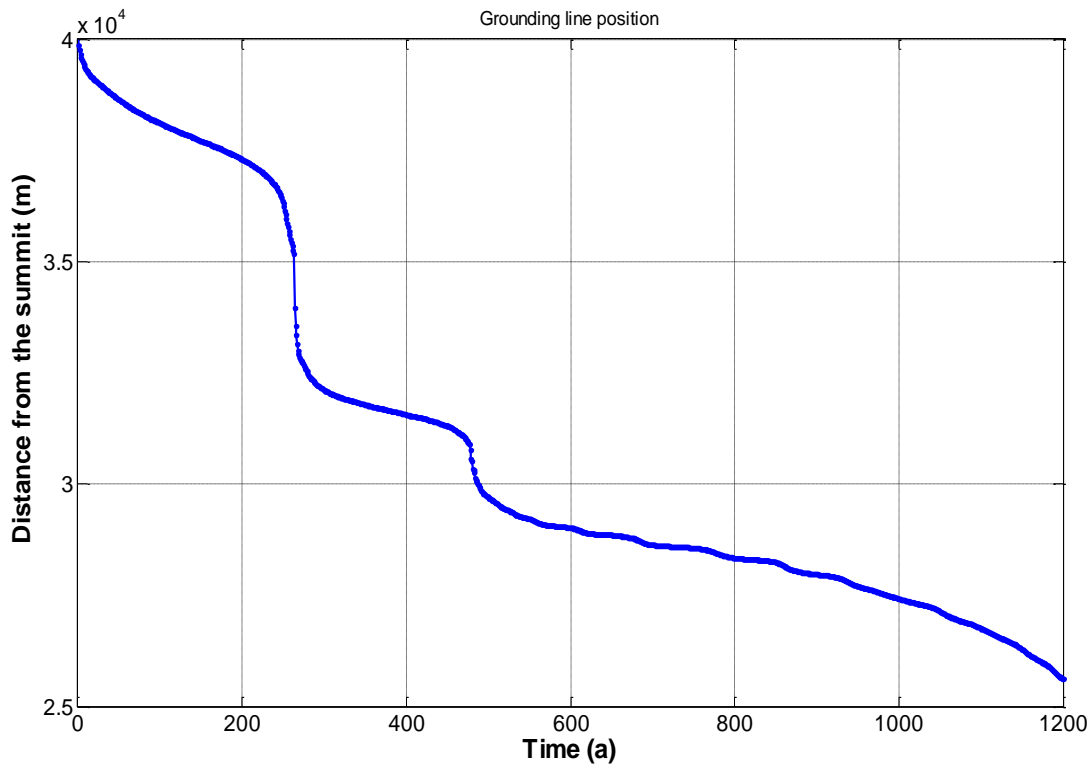
The assessments of inflow of mass from the regions surrounding the ice streams show that the possible ingoing ice mass is sufficiently smaller than the outgoing to the ocean ice flow or inflows/outflows of the ice mass from the ice surface. Thus, accounting of the ingoing from the lateral edges ice mass should not essentially impact on the conclusion about the continuous ice decline along the flow lines.

**Referee #2:** *“The authors use a constant present-day mass balance, while state-of-the-art modelling involves the use of projected changes in climatic conditions (e.g., Winkelmann et al., 2012; Clarke et al., 2015). This would give the paper significantly more substance and allow to interpret the results in context of the presently observed changes in climate.”*

We used the constant present-day mass balance with intent to show that even if the future warming is not accounted in the surface mass balance, the modelling reveals the continuous ice decline along the flow lines over long time scales. Moreover, the steady state is not achieved for the reference surface mass balance (see, for instance, Fig. 10A, Fig. 11A).



**Fig. 10A.** The modeled successive B-B' cross-section geometries separated by 120-year intervals from the present to the future 1200 years later.



**Fig. 11A.** The modeled grounding line history for B-B' cross section

Authors thank the referee#2 for the specific comments.

Thanks and all the best,

Yuri V. K.