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Supplement to reply to Shaun Lovejoy

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Results of multifractal analysis

The exponent β is well-defined as long as the power spectral density function $S(f)$ is a power law in f, or equivalently if the wavelet fluctuation function $E|W(t,\Delta t)|^2$ is a power law in Δt. If well-defined, the β exponent is related to the temporal correlations in the signal via simple formulas. In fact, for a (zero-mean) stationary process $T(t)$ with $-1<\beta<1$ we have $\langle T(t)T(t+\Delta t)\rangle \sim (\beta+1)\beta\Delta t^{\beta-1}$ and for (a zero-mean) process with stationary increments and $1<\beta<3$ we have $\langle \Delta T(t)\Delta T(t+\Delta t)\rangle \sim (\beta-1)(\beta-2)\Delta t^{\beta-3}$, where $\Delta T(t)$ is the increment process of $T(t)$ (Rypdal and Rypdal, 2012). Thus, the results presented so far in this paper do not rely on any assumptions of self-similar or multifractal scaling. It is only assumed that the second-order fluctuation functions $\langle |W(t,\Delta t)|^2\rangle$ are well approximated by power-laws over an extended range of time scales.

A more complete scaling analysis can be performed if one imposes the more restrictive assumption that the wavelet-based structure functions $\langle |W(t,\Delta t)|^q\rangle$ are power-laws in Δt, not only for $q=2$, but for an interval of q-values. It is then it is possible to define a scaling function $\tau(q)$ via the relation

$$\langle |W(t,\Delta t)|^q\rangle \sim \Delta t^{\tau(q)}.$$

We observe that $\tau(2)=\beta$. If $T(t)$ is self-similar (or if $T(t)$ is the increment process of a self-similar process in the case $\beta<1$) we have $\tau(q)=\beta q/2$, but in general, the $\tau(q)$ may be concave. Processes that exhibit power-law structure functions and strictly concave scaling functions can be characterized as multifractal intermittent.

If Fig. 1 we present a multifractal analysis of the data sets considered in this paper using q-values in the range from 0.1 to 4. For the Holocene we find linear scaling functions for both the instrumental record and the Moberg Northern Hemisphere reconstruction, and in the NGRIP data we find linear scaling functions for the both the stadial periods and the interstadial periods when these are analysed separately, although, as we have already seen, there is a deviation from the $1/f$ scaling in the stadial periods for time scales shorter than about 200 yrs. If the NGRIP record is analysed with both stadial and interstadial stages included, then it is not clear how to define the scaling function since the shifts between the two types of stages causes a “break” in the power-law scaling of the wavelet-based structure functions. If we define $\tau(q)$ using the time scales shorter than 2 kyr we obtain a linear scaling function corresponding to $\beta=1.14$, and if we use the time scales longer than 4 kyr we obtain a linear scaling function corresponding to $\beta = 1.78$. In neither case do we obtain a strictly concave scaling function. A linear scaling function is also obtained if we disregard the “break” in the scaling and fit power laws using all the available time scales. In this case the scaling function corresponds to $\beta = 1.26$. For the periods of the EPICA record that corresponds to ice ages, we find wavelet-based structure functions that are closer to power-laws than what is observed in the NGRIP record. This is expected since the abrupt transitions between cold and warm periods is much less pronounced in Antartica than in Greenland. The scaling function for the ice-age periods in the EPICA data is linear and corresponds to $\beta = 1.18$.

The results discussed above show that there is no evidence of multifractal intermittency in the temperature records analysed in this paper. This is not very surprising and could be established by direct inspection of the data record. The trained observer would use the fact that if $\tau(q)$ is strictly concave, then the kurtosis of $W(t,\Delta t)$,

$$\frac{\langle |W(t,\Delta t)|^4\rangle}{\langle |W(t,\Delta t)|^2\rangle^2} \sim \Delta t^{\tau(4)-2\tau(2)},$$

is decreasing as a power-law function of Δt, and is therefore leptokurtic[1] on the shorter time scales Δt. Multifractal intermittency also implies that the amplitudes of the random fluctuations are clustered in time, on all time scales, as observed in intermittent turbulence or financial time series (see e.g., Bouchaud and Muzy (2003)). These are *not* prominent features in the time series analysed in paper. For the NGRIP data, the δ^{18}O ratio slightly deviates from a normal distribution as a result of the DO events, but this is not well described by a multifractal model since that would require the wavelet-based structure functions to be power-laws in Δt. In fact, what we show in this paper is that effect of DO events is to break the scaling, rather than to produce multifractal scaling.

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[1]A distrubution is leptokurtic if it has high kurtosis compared with a normal distribution. This means that the probability density function has a high central peak and fatter tails.

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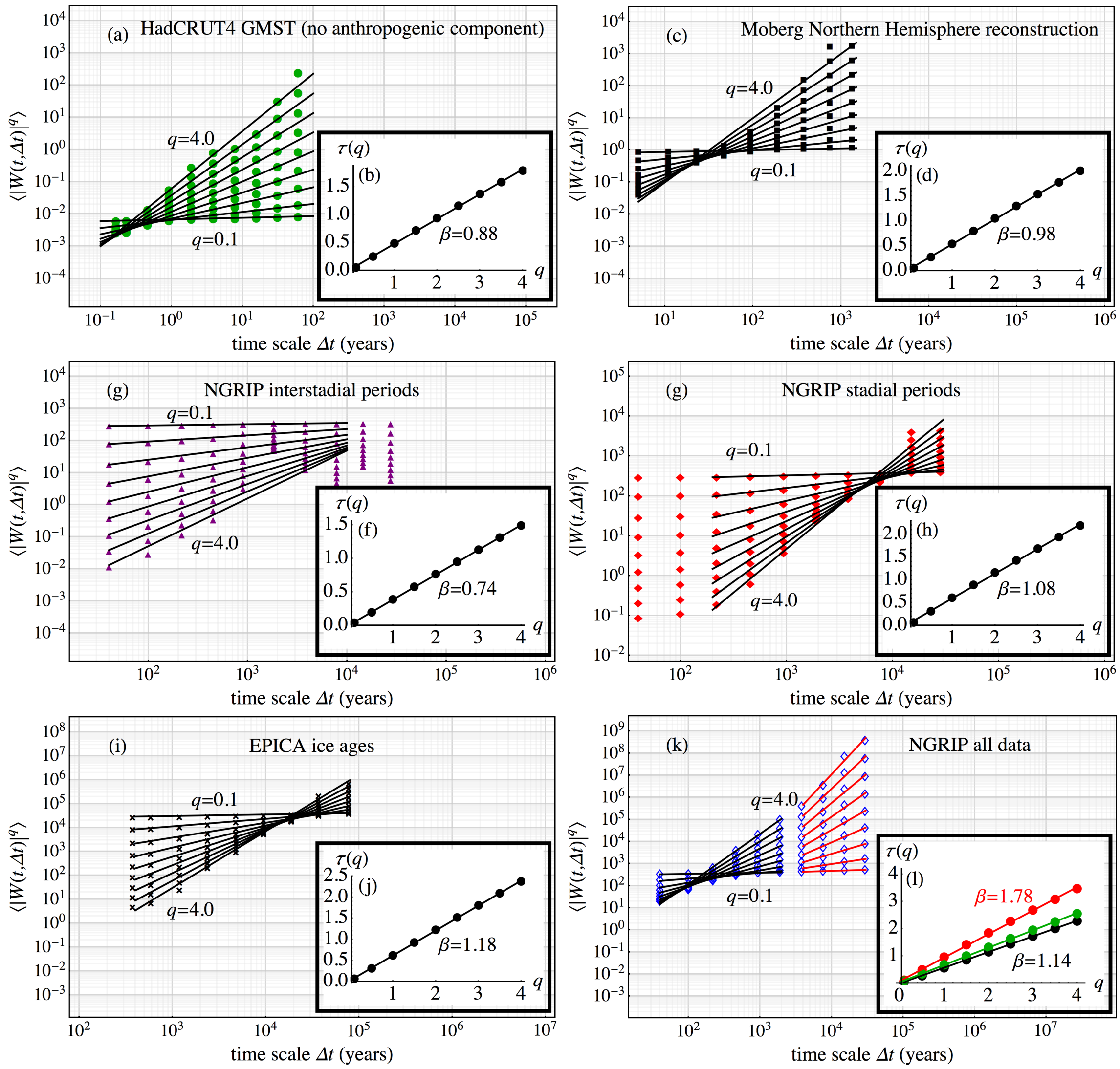

Figure 1. (a): The estimated wavelet-based structure functions $\langle |W(t,\Delta t)|^q \rangle$ for the HadCRUT4 monthly global mean surface temperature where the anthropogenic component has been removed using a linear-response model. The lines show the fitted power-law functions $c_q \Delta t^{\tau(q)}$. The q-values are $q = 0.1, 1.0, 1.5, \ldots, 4.0$. (b) The scaling function $\tau(q)$ obtained from the fitted power-laws in (a). The line is a linear fit to the estimated scaling function, and the slope of this line is $\beta/2$ with $\beta = 0.88$. (c-d): As (a) and (b) but in this case for the Moberg Northern Hemisphere reconstruction. (g-f):