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Long-run evolution of the global economy: 2. Hindcasts of innovation and growth

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Abstract

Long-range climate forecasts rely upon integrated assessment models that link the global economy to greenhouse gas emissions. This paper evaluates an alternative economic framework, outlined in Part 1, that is based on physical principles rather than explicitly resolved societal dynamics. Relative to a reference model of persistence in trends, model hindcasts that are initialized with data from 1950 to 1960 reproduce trends in global economic production and energy consumption between 2000 and 2010 with a skill score greater than 90 %. In part, such high skill appears to be because civilization has responded to an impulse of fossil fuel discovery in the mid-twentieth century. Forecasting the coming century will be more of a challenge because the effect of the impulse appears to have nearly run its course. Nonetheless, the model offers physically constrained futures for the coupled evolution of civilization and climate during the Anthropocene.

1 Introduction

Climate simulations require as input future scenarios for greenhouse gas emissions from Integrated Assessment Models (IAMs). IAMs are designed to explore how best to optimize societal well-being while mitigating climate change. The calculations of human behaviors are made on a regional and sectoral basis and can be quite complex, possibly with hundreds of equations to account for the interplay between human decisions, technological change, and economic growth (Moss et al., 2010; IPCC, 2014).

Periodically, model scenarios are updated to account for observed emissions trajectories. It has been noted that the global carbon dioxide (CO_2) emission rate has not only grown along a "business-as-usual" (BAU) trajectory, but has in fact slightly exceeded it (Raupach et al., 2007; Peters et al., 2013), in spite of a series of international accords aimed at achieving the opposite (Nordhaus, 2010).

Such inertia might suggest that the human system is behaving much like a physical system. It tends to be the slowest and largest aspects of current variability that are the most powerful and least responsive. Slow variations represent an accumulated history of frequent, small-scale events. Current events have a correspondingly limited impact on the future because they become diluted in all actions that preceded them (Hasselman, 1976). One reason we continue to use fossil fuels today is that we have spent at least a century accumulating a large global infrastructure to do so. It is not that current efforts to move civilization towards renewables cannot change this trajectory of carbon dependency, but rather that it will take considerable effort and time.

Inertia offers plausibility to a business-as-usual trajectory, particularly for something as highly integrated in the space and time as CO_2 emissions by civilization as a whole. Still, assuming persistence in trends is something that should only be taken so far. By analogy to meteorological forecasts, it is reasonable to assume that clearing skies will lead to a sunny day. However, prognostic weather models are based on fundamental physical principles that tell us that it cannot keep getting sunnier. Even a very simple set of equations dictates that at some point a front will pass, clouds will form, and a high pressure system will decay.

So, it is by getting the underlying physics right that we are able to achieve some level of *positive skill* in any forecast attempt (Fig. 1). Part 1 of this study describes a physical model that provides expressions for making long-range economic forecasts of civilization evolution (Garrett, 2014). Much like the primitive equations of a prognostic weather model, global economic growth is expressed as a non-equilibrium response to external gradients driving energy dissipation and material flows. The model differs from IAMs by including no explicit role for human decisions; the physics does not allow for mathematical expressions of policy. Rather, economic innovation and growth is treated primarily as a geophysical phenomenon, in other words as an emergent response to available reserves of raw materials and energy supplies.

This is a much more deterministic approach than traditional approaches that appeal more to policy based scenarios. As an alternative, it offers a means for integrating

human systems and physical systems under a common framework, one where the governing equations are consistently derived from first thermodynamic principles.

Whether the approach that is applied is rooted in policy or physics, it is important that any societal model be evaluated for performance. Weather, climate, and financial models are regularly evaluated through hindcasts or backtesting. Economic models that simulate the long-run development of humanity need not be an exception. A good model, even one that includes policy, should be able to reproduce current events with positive skill starting at a some point decades in the past. To beat the zero-skill hindcast of persistence, the model would invoke fits to concurrent trends to the minimum extent possible.

In the period following World War II, an economic "front" passed that propelled civilization towards unprecedented levels of prosperity and, by proxy, greenhouse gas emissions. This paper examines whether the theoretical model introduced in Part 1 can explain the evolution of this front. Section 2 outlines the philosophical and thermodynamic basis for describing economic evolution with physics. Section 3 evaluates this model from hindcasts. Sections 4 and 5 discuss and summarize the results.

2 Forces for economic growth

2.1 Reversible cycles and irreversible flows

The macroeconomic component of IAMs considers labor, physical capital and technological change to be the motive forces for economic production and growth. The focus is on individuals, nations, and economic sectors. The model equations describe how physical capital and human prosperity grow with time, and how energy choices tie in with greenhouse gas emissions (Solow, 1956; Nordhaus and Sztorc, 2013).

There is a second approach which is to consider that CO_2 is long-lived and well-mixed in the atmosphere, so the magnitude of greenhouse forcing is almost entirely unrelated to the national origin of anthropogenic emissions. Then, civilization can be

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described as a whole, one where small-scale details at personal, regional, or sectoral levels are not treated explicitly. The only quantity that is resolved is an aggregated global economy that is inclusive of all civilization elements, including human and physical capital combined.

An implicit consideration with this approach is a separation of small, short-term, “micro” economic behaviors from larger, longer-term, “macro” economic evolution. From the perspective of thermodynamics, short-term equilibrium reversible, cyclic behaviors that are not explicitly resolved are separated from longer-term non-equilibrium irreversible dynamics that are resolved. This is a common strategy, one illustrated in Fig. 2, as familiar as the separation of the tachometer and speedometer in a car. One represents reversible engine cycles, whereas the other expresses the rate of irreversible travel down the road.

In general, reversible and irreversible processes are linked. This is because the Second Law prescribes that all processes are irreversible. Introducing the concept of reversible circulations within a system is a useful idealization. However, such circulations can only be sustained by an external, irreversible flow of energy and matter through the system. When open systems are near a balance or a steady-state, then reversible circulations can be represented as a four step Carnot cycle whereby external heating raises the system potential so that raw materials diffuse from outside the system to inside the system (Zemanksy and Dittman, 1997). Waste heat is dissipated to the environment so that the system can relax to its ground potential state where it releases exhaust or undergoes decay. Averaged over time, the circulations within the system maintain a fixed amplitude and period τ_{circ}.

For example, the dynamic circulations of a hurricane are sustained by a inflow of oceanic heat and an outflow of thermal radiation to space (Emanuel, 1987). In the case of civilization, we consume energy in order to sustain circulations and extract raw materials from the environment, leaving behind material waste and radiated heat. Petroleum in a car propels our material selves to and from work where we consume carbohydrates, proteins and fats to propel electrical signals to and from our brains so

that we can consume electricity from coal in order to propel charge along copper wires to and from our computers. Through radiation, frictional losses, and other inefficiencies, all potential energy is ultimately dissipated as waste heat to the atmosphere and ultimately through radiation to space at the mean planetary blackbody temperature of 255 K. Over short timescales, consumption approximately equals dissipation, and civilization circulations maintain a steady-state.

Over longer timescales, any small imbalance between consumption and dissipation by civilization becomes magnified. Raw materials are slowly incorporated into civilization's fabric at rate $1/\tau_{\text{growth}}$ (Fig. 2). Further, at the same time that civilization grows, resources are discovered and depleted, and perhaps as a consequence of climate change, decay rates increase too. The focus shifts from the short-term reversible circulations associated with our daily lives to the longer term timescales associated with the non-equilibrium, irreversible growth of civilization as a whole.

2.2 The relationship of energy dissipation to human wealth

A formal framework for the non-equilibrium thermodynamics of civilization growth was laid out in Part 1 (Garrett, 2014). The connection to economic growth in monetary terms was an identity linking an expression of wealth to how fast civilization dissipates energy (Garrett, 2011). We all have some sense that civilization can be distinguished from its uncivilized surroundings through the existence of our farms, buildings, human population, vehicles, and communication networks. This distinction implies a gradient where gradients are associated with a consumption and dissipation of potential energy. Absent energy consumption, civilization would necessarily decay towards an uncivilized equilibrium where the gradient ceased to exist and all internal circulations stopped.

The hypothesis that was made is that the capacity to sustain circulations through energy dissipation has a value that can be assigned economic units of currency. New, or added value is produced when work is done by way of economic production to expand the capacity to dissipate energy. Real production occurs when raw materials are incorporated into civilization's structure at a net positive rate to enlarge civilization, thereby creating new wealth and increasing its capacity to consume. Expressed as an integral, it is the net accumulation of real production that yields current economic wealth. Current economic wealth determines our current capacity to consume energy in order to sustain existing circulations.

The analytical formulation of these statements is that instantaneous power dissipation, or the rate of primary energy consumption a by all of civilization (units power), is linked through a constant λ (units power per unit currency) to civilization's inflation-adjusted economic value (or civilization wealth) C (units currency). Wealth is an accumulation of the Gross World Product (GWP) Y, adjusted for inflation at market exchange rates (MER) (Garrett, 2011)

$$a = \lambda C = \lambda \int_0^t Y(t')\,dt' \tag{1}$$

Alternatively, and taking the derivative with respect to time, economic production is a representation of a growth in the capacity to consume energy that might be obtained through a convergence of raw materials:

$$\frac{dC}{dt} = Y \tag{2}$$

where, since $a = \lambda C$, the production function is given by

$$Y = \frac{1}{\lambda}\frac{da}{dt} \tag{3}$$

The argument is that there is no intrinsic wealth in and of itself. As discussed in Part 1, wealth is built from a network of connections. Connections are what enable dissipative flows insofar as there exist potential energy gradients to drive the flows. For civilization as a whole, wealth is sustained by primary power consumption through the connections we have to reserves of fossil, nuclear, and renewable energy sources.

Within civilization, wealth is due to the connections between and among ourselves and our "physical capital". All aspects of civilization, whether social or material, compete for globally available potential energy. Financial expressions of any element's value reflect the relative extent to which its connections enable civilization to sustain global scale circulations and wealth.

Wealth emerges from the past through prior production of these connections. Value added to civilization through the construction of a house decades ago still contributes to value today, provided the house remains part of a network with ties it to the remainder of civilization. Even with no one home and all the utilities turned off, a house maintains its value for as long as it can be perceived by other active members of the global economy.

Equations (1) to (3) are unorthodox by traditional economic standards. Wealth is more usually expressed in terms of physical capital, or as a stock that has an intrinsic value. New capital is produced using currently existing labor and capital. Production levels are assumed to be independent of external physical constraints (Solow, 1956).

However, there is an another line of economic thought that has pointed out that a the consumption and dissipation of potential energy must be required for economic production (Lotka, 1922; Soddy, 1933; Odum, 1971; Georgescu-Roegen, 1993; Hua and Bakshi, 2004; Annila and Salthe, 2009). Attempts to establish a more explicit link between physical and financial quantities have noted that there exists a high correlation between national or sectoral economic production and energy consumption (Costanza, 1980; Cleveland et al., 1984; Brown et al., 2011). Alternative economic growth models partially substitute energy for labor and capital as a productive force (Ayres and Warr, 2009; Kümmel, 2011).

The model presented here follows in this line of inquiry although it differs in several key regards. First, by standing back, civilization is examined only as a whole. While nothing can be said about complications associated with internal trade, they also do not need to be considered within this framework because they are not explicitly resolved. Moreover, no distinction is made between human and physical capital: energy consumption is a complete substitute for both these quantities. Most fundamentally, the model is strictly thermodynamic, in which case no requirement exists for dimensionally inconsistent fits to prior economic data that are dependent on the time and place that is considered. The model does not rest upon any statistical correlation between energy consumption and economic production (something that has has been erroneously claimed by others Cullenward et al., 2011; Scher and Koomey, 2011). Instead, the model's validity rests on the existence of a fixed ratio between energy consumption a and the time integral of inflation-adjusted economic production C (Eq. 1). The perspective is simply that current energy consumption and dissipation sustains all of civilization's circulations, insofar as they have accumulated through prior economic growth.

What matters most is that this hypothesis is falsifiable based on available data. Expressing a in units of Watts, and Y in units of 2005 MER US dollars per second, then wealth has units of 2005 MER US dollars, and the constant λ has units of Watts per 2005 MER US dollar. What was shown in Table S2 of Garrett (2014), and in graphical form in Fig. 3 is that, for the period 1970 to 2010 for which global statistics for power consumption are available, the mean value of λ is 7.1 milliwatts per 2005 US dollar. GWP more than tripled over this time period. From year to year, the SD in the ratio $\lambda = a/C$ is just one percent, implying an uncertainty in the mean at the 95 % confidence level of 0.1 milliwatts per 2005 US dollar. This theoretical and empirical support for λ being effectively a constant is the basis for expressing real global economic wealth as a circulation that is sustained by physical flows.

2.3 Past economic innovation as the engine for current economic growth

The existence of a constant value for λ indicates that economic wealth cannot be decoupled from energy consumption. For the past, reconstructions of global rates of energy consumption going back 2000 years are provided in Table S3 of Garrett (2014). For the future, CO_2 emissions will be inextricably linked to global prosperity for as long as the economy relies on fossil fuels (Garrett, 2011). Increasing energy efficiency may

be a commonly supposed mechanism for reducing energy consumption while maintaining wealth. However, as elaborated in Appendix A, this does not appear to be the case.

The basic reason is that, from Eq. (1), the growth rate of civilization wealth C and its rate of energy consumption a are equivalent:

$$\text{rate of return} = \eta = \frac{\mathrm{d}\ln a}{\mathrm{d}t} = \frac{\mathrm{d}\ln C}{\mathrm{d}t} \tag{4}$$

Effectively, like interest on money in the bank, the parameter η represents the "rate of return" that civilization enjoys on its current wealth C, and that it sustains by consuming ever more power.

Since 1970, average rates of return for a and C have been 1.90 % per year (Fig. 3). Substituting Eq. (1) into Eq. (4) yields a relationship between the rate of return and the inflation-adjusted GWP:

$$Y = \eta C = \eta \int_0^t Y(t')\,\mathrm{d}t' \tag{5}$$

or,

$$\eta = \frac{Y}{\int_0^t Y(t')\,\mathrm{d}t'} \tag{6}$$

So, the current rate of return is tied to past, since it expresses the ratio of current real production to the historical accumulation of past real production.

The rate of change of civilization's rate of return can be referred to as an "innovation rate"

$$\text{innovation rate} = \mathrm{d}\ln\eta/\mathrm{d}t \tag{7}$$

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Referring to an acceleration term $\mathrm{d}\ln\eta/\mathrm{d}t$ this way might seem a bit arbitrary. However, in Appendix A it is shown that it corresponds directly to more traditional economic interpretations such as an increase in the "total factor productivity" or the "production efficiency". For example, it is easy to show from Eqs. (1) and (5) that $\eta = \lambda Y/a$. Since λ is a constant, increases in the production efficiency (or inverse energy intensity) Y/a are equivalent to the expression for innovation $\mathrm{d}\ln\eta/\mathrm{d}t$. Innovation is a driving force for economic growth since it follows directly from Eqs. (4) and (5) that the real GWP growth rate is governed by the relationship

$$\frac{\mathrm{d}\ln Y}{\mathrm{d}t} = \eta + \frac{\mathrm{d}\ln\eta}{\mathrm{d}t} \tag{8}$$

GWP growth = rate of return + innovation rate

The rate of return η is equivalent to the time-integral of past innovations through $\eta = \int_0^t \mathrm{d}\eta/\mathrm{d}t'\mathrm{d}t'$, and so Eq. (8) can be expressed as

$$\frac{\mathrm{d}\ln Y}{\mathrm{d}t} = \int_0^t \left(\mathrm{d}\eta/\mathrm{d}t'\right)\mathrm{d}t' + \frac{\mathrm{d}\eta/\mathrm{d}t}{\int_0^t \left(\mathrm{d}\eta/\mathrm{d}t'\right)\mathrm{d}t'} \tag{9}$$

The implication here is that current rates of GWP growth can be considered to be a consequence of past innovations (the first term) and current innovations insofar as they are not diluted by past innovations (the second term). Accordingly, current GWP growth rates will tend to persist; new technological advances will always struggle to replace older advances that are already in place (Haff, 2014). Placing an internal combustion engine on a carriage was revolutionary for its time, but only a series of more incremental changes have been made to the concept since. Any new dramatic change has to compete with the large vehicular infrastructure that has already been put in place.

Figure 4 shows how rates of return, innovation rates, and GWP growth have been changing in recent decades. The rate of return η generally has had an upward trend.

In 2008, the rate of return on global wealth reached an all time historical high of 2.24 % per year, up from 1.93 % per year in 1990 and 0.71 % per year in 1950.

Meanwhile, innovation rates have declined. The rate of growth of the rate of return, or $\mathrm{d}\ln\eta/\mathrm{d}t$ has dropped from around 4 percent per year in 1950 to near stagnation today. Unprecedented gains in production efficiency that were obtained in the two decades after the Second World War appear to have since given way to much more incremental innovation.

From Eq. (8), GWP growth is the sum of these two pressures. On one hand, positive innovation has had a lasting positive impact on the GWP since it has led to an ever increasing rate of return η. On the other hand, innovation rates have declined. Between 1950 and 1970, GWP growth rates were between four and five percent per year. Since 1980, they have been closer to three percent per year. Increasingly, the long-term increase in civilization's rate of return η has been offset by the long-term decrease in innovation $\mathrm{d}\ln\eta/\mathrm{d}t$. The turning point was in the late 1970s when, as shown in Fig. 4, innovation rates dipped below rates of return. Between 1950 and 1975, current innovation was the largest contributor to current GDP growth rates. Since then, continued GDP growth has relied increasingly on innovations made in the first two decades since the end of World War II (Eq. 9).

2.4 Forces for innovation

For the special case shown in Fig. 2 that there is positive net convergence of matter in the system, the system grows. It extends its interface with accessible reserves of energy and matter. An enlarged interface allows for faster rates of consumption. The result is a positive feedback that allows growth to accelerate. This is a basic recipe for emergent or exponential growth. One important aspect of this feedback, however, is that rates of exponential growth are never constant. Rather, they increase when new reserves of energy or matter are discovered and they decrease when there is is accelerated decay.

The thermodynamics of this recipe (Garrett, 2012b) were applied in Part 1 to the emergent growth of civilization and its rates of return on wealth (Garrett, 2014). It was shown that the rate of return η can be broken down into the proportionality

$$\eta \propto (1-\delta)\,\frac{\Delta H_\mathrm{R}}{N_\mathrm{S}^{2/3} e_\mathrm{S}^\mathrm{tot}} \tag{10}$$

Here, N_S represents the accumulated material size of civilization due to past production. δ is a decay parameter that accounts for how rapidly N_S falls apart due to natural causes. ΔH_R represents the size of the energy reserves that are available to be consumed by civilization. The term $e_\mathrm{S}^\mathrm{tot}$ represents how much of this energy must be consumed by civilization in order to incorporate raw materials into civilization's fabric, thereby adding to N_S. The exponent $2/3$ arises from how flows are down a gradient and across an interface.

Building on the identity $a = \lambda C$, it was argued that rates of economic innovation can be represented by

$$\frac{\mathrm{d}\ln\eta}{\mathrm{d}t} = -2\eta + \eta_\mathrm{tech} \tag{11}$$

innovation rate = diminishing returns + technological change

The first term represents a drag on innovation due to a law of diminishing returns -2η. The second term expresses a rate of technological change η_tech due to changes in δ, ΔH_R and $e_\mathrm{S}^\mathrm{tot}$. A distinction is made here between a technological advance and an innovation. Technological change only counts as an innovation if it overcomes diminishing returns to lead to a real increase in the rate of return η.

A law of diminishing returns is a characteristic feature of emergent systems. As indicated by Eq. (10), the exponential growth rates of larger, older objects tend to be lower than for smaller, younger ones. In our case, our bodies are a complex network of nerves, veins, gastro-intestinal tracts and pulmonary tubes. We use this network so

that we can interact with a network of electrical circuits, communication lines, plumbing, roads, shipping lanes and aviation routes (van Dijk, 2012). Such networks have been built from a net accumulation of matter. So, as civilization grows, any given addition becomes increasingly incremental.

The implication of Eq. (11) is that, absent sufficiently rapid technological change, relative growth rates η will tend to decline, and innovation will turn negative. For example, from Eq. (11), innovation requires that $\eta_\mathrm{tech} > 2\eta$. Or, by substituting Eq. (8) into Eq. (11), an expression for GWP growth is $\mathrm{d}\ln Y/\mathrm{d}t = -\eta + \eta_\mathrm{tech}$, in which case maintenance of positive GWP growth requires that $\eta_\mathrm{tech} > \eta$. That economic growth has been sustained over the past 150 years is a testament to to the importance of technological change for overcoming diminishing returns.

The rate of technological change follows from Eq. (10):

$$\eta_\mathrm{tech} = \frac{\mathrm{d}\ln(1-\delta)}{\mathrm{d}t} + \frac{\mathrm{d}\ln\Delta H_\mathrm{R}}{\mathrm{d}t} - \frac{\mathrm{d}\ln e_\mathrm{S}^\mathrm{tot}}{\mathrm{d}t} \tag{12}$$

$$\eta_\mathrm{tech} = \eta_\delta + \eta_\mathrm{R}^\mathrm{net} + \eta_\mathrm{e}$$

technological change = improved longevity + net reserve discovery
+ extraction efficiency gains

The first of these three forces is improved longevity. Supposing that civilization decays at rate j_d and incorporates new matter at rate j_a. A dimensionless decay parameter $\delta = j_\mathrm{d}/j_\mathrm{a}$ expresses the drag on civilization's material growth due to decay: if δ were zero, there would be no offset to new network growth. Any decline in δ would therefore represent an increase in civilization's longevity. For example, δ might decrease as civilization shifts from wood to steel as a construction material. Or, it might increase due to more frequent natural disasters from climate change.

In Part 1, it was shown how the nominal GWP can be tied to j_a, and the real inflation-adjusted GWP can be tied to $j_\mathrm{a} - j_\mathrm{d}$. At domestic scales, the so-called "GDP deflator" is often used as an analog for the annual inflation rate $\langle i \rangle$ since it represents the fractional

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downward adjustment that is imposed on the nominal GDP to obtain the real GDP. For civilization as a whole, the implication is that declining decay, or increased longevity, corresponds with declining inflation and faster real GWP growth, i.e.

$$\eta_\delta \simeq \frac{\mathrm{d}\langle\delta\rangle}{\mathrm{d}t} \simeq -\frac{\mathrm{d}\langle i\rangle}{\mathrm{d}t} \tag{13}$$

The second force for technological change in Eq. (12) is discovery of new energy reserves. Where it exceeds reserve depletion, it accelerates economic innovation through an increase in the size of available energy reserves ΔH_R (Smil, 2006; Ayres and Warr, 2009). Energy reserves decline as they are consumed at rate a. Meanwhile civilization discovers new reserves at rate D. The rate of net discovery is

$$\eta_\mathrm{R}^\mathrm{net} = \frac{D-a}{\Delta H_\mathrm{R}} \tag{14}$$

Provided that reserves expand faster than they are depleted, then the rate $\eta_\mathrm{R}^\mathrm{net}$ is positive. It represents a technological advance because there is reduced competition for available resources.

The specific enthalpy of civilization $e_\mathrm{S}^\mathrm{tot}$ in Eq. (12) is an expression of the amount of power a that is required for civilization to extract raw materials and to incorporate them into civilization's fabric at rate j_a. If the ratio a/j_a declines, then civilization becomes more energy efficient.

For example, mining and forestry is currently powered by large diesel engines rather than human and animal labor. Civilization is able to extract raw materials with comparative efficiency and lengthen civilization networks at a corresponding greater rate. Using less energy, we are able to build more roads, lengthen communications networks, and even increase population, as we too are made of matter and are part of civilization's fabric. Where the extraction efficiency of raw materials improves, it is an effective force for technological change defined by

$$\eta_\mathrm{e} = \frac{\mathrm{d}\ln j_\mathrm{a}}{\mathrm{d}t} - \frac{\mathrm{d}\ln(a)}{\mathrm{d}t} \tag{15}$$

2.5 Deterministic solutions for economic growth

Equation (11) for innovation is logistic in form. That is, it could be expressed as $\mathrm{d}\eta/\mathrm{d}t = \eta_\mathrm{tech}\eta - 2\eta^2$ with a rate of exponential growth η_tech and a drag rate on growth of -2η. An initial exponential growth phase yields to diminishing returns where growth rates stabilize (Fig. 5). If η_tech is constant, then the solution for the rate of return η is

$$\eta(t) = \frac{\eta_\mathrm{tech}/2}{1+(G-1)\exp(-\eta_\mathrm{tech}t)} \tag{16}$$

where

$$G = \frac{1}{2}\frac{\eta_\mathrm{tech}}{\eta_0} \tag{17}$$

represents a "Growth Number" (Garrett, 2012b, 2014) and the subscript 0 indicates the initial observed value for η_0. The solution for η in Eq. (16) is sigmoidal. Provided G is greater than one, then rates of return initially increase exponentially and saturate at a rate of $\eta_\mathrm{tech}/2$. So, for example, if η_tech is sustained at 5 % per year, then one would expect rates of return to grow sigmoidally towards 2.5 % per year. The characteristic time for the exponential growth phase would be $1/\eta_\mathrm{tech}$, or 20 years.

From Eq. (8), the corresponding time-dependent solution for GWP growth assuming a fixed rate of technological change is

$$\frac{\mathrm{d}\ln Y}{\mathrm{d}t}(t) = \frac{\eta_\mathrm{tech}}{2}\left[\frac{1+2(G-1)\exp(-\eta_\mathrm{tech}t)}{1+(G-1)\exp(-\eta_\mathrm{tech}t)}\right] \tag{18}$$

Here, GWP growth rates also saturate at a value of $\eta_\mathrm{tech}/2$, but if $G>1$ then this is by way of decline rather than growth. Thus, rates of return on wealth (Eq. 16) and rates of GWP growth (Eq. 18) should have a tendency to converge with time. This is in fact precisely the behavior that has been observed in the past few decades. Figure 4

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shows values of η and $d\ln Y/dt$ that differ by about a factor of four in 1950 but that are approaching from opposite directions towards a common value of about 2.5 % per year.

3 Model validation through hindcasts

Three approaches here are taken to evaluate the validity of Eq. (11) for expressing the long-run evolution of the global economy.

3.1 The functional form relating innovation to growth

Figure 6 shows the relationship between innovation rates and rates of return over the past three centuries (see Part 1 for associated statistics). Rapid innovation and accelerating rates of return characterized the industrial revolution and the late 1940s. Periods of subsiding innovation followed 1910 and 1950.

Equation (11) implies that, assuming that η_{tech} is a constant, innovation rates $d\ln\eta/dt$ should be related to rates of return η by a slope of -2; the intercept should be equivalent to the rate of technological change η_{tech} given by Eq. (12). For the period since 1950 where statistical reconstructions of GWP are yearly and presumably most reliable Maddison (2003), Fig. 6 shows that the past 60 years have been characterized by a least-squares fit relationship between innovation rates $d\ln\eta/dt$ and rates of return η (with 95 % uncertainty bounds) given by

$$\frac{d\ln\eta}{dt} = -(2.54 \pm 0.54)\,\eta + (0.06 \pm 0.01) \tag{19}$$

Within the stated uncertainty, the observed slope relating innovation to rates of return is consistent with the theoretically expected value of -2 that comes from a law of diminishing returns. The implied rate of technological discovery for this time period η_{tech} is the intercept of the fit, or about 6 % per year. The magnitude of the difference of the

fit from the anticipated slope might be an indication that η_{tech} has not been constant, but has declined with time as discussed below.

3.2 Hindcasts of long-run civilization growth

Approaching the problem as a hindcast, a hypothetical economic forecaster in 1960 might have noted that the average values of η and $d\ln\eta/dt$ between 1950 and 1960 were 0.9 % per year and 3.3 % per year, respectively. From Eq. (11), this implies that η_{tech} was 5.1 % per year during this period. Applying Eqs. (16) and (18), and using an initial value for η_0 of 1.0 % per year in 1960, the forecaster could then have obtained the trajectories for economic innovation and growth that are shown in Fig. 7.

50 year hindcasts are summarized in Table 1 along with skill scores defined relative to a reference model of persistence in trends (American Meteorological American Meteorological Society, 2014):

$$\text{Skill Score} = 1 - \frac{|\text{Error(hindcast)}|}{|\text{Error(persistence)}|} \tag{20}$$

Skill scores are positive when the hindcast beats persistence in trends, and zero when they do not. For example, average rates of energy consumption growth in the past decade would have been forecast to be 2.3 % per year relative to an observed average of 2.4 % per year. Relative to a persistence prediction of 1.0 % per year, the skill score is 96 %. Or, a forecast of the GWP growth rate for the first decade of this century would have been 2.8 % per year compared to the actual observed rate of 2.6 % per year. The persistence forecast based on the 1950 to 1960 period is 4.0 % per year, so the skill score is 91 %.

3.3 Observed magnitude of technological change

High skill scores suggest that it is possible to provide physically constrained scenarios for civilization evolution over the coming century. There do not appear to be other macro-economic forecast models that are equally successful. Macro-economic models are not normally evaluated through comparisons to multi-decadal historical data. Where they are, it is not in the form of a true hindcast. Rather, what is examined is the extent to which a sufficiently complex production function can be tuned to provide an accurate fit to prior observations (e.g., Warr and Ayres, 2006).

Ideally, however, not even the value of η_{tech} would be derived from a fit in the model presented here, even if it is to data prior to the date of model initialization. Although a fixed value for η_{tech} appears to work well for the purpose of making hindcasts, future rates of technological change can obviously be expected evolve with time.

To this end, an attempt has been made here to quantify the thermodynamic forces outlined in Eq. (12). Methods for estimating a time series for the sizes of energy reserves, the rate of energy consumption, the rate of raw material consumption, and economic inflation during the period between 1950 and 2010 are described in Appendix B and are summarized in Table 2. Average rates are shown for three successive 20 year periods beginning in 1950, and for the 1950 to 2010 period as a whole.

What stands out is how there was unusually rapid technological change between 1950 and 1970. This period was characterized by rapidly growing access to reserves of oil, gas, and raw materials. It was followed by an abrupt slowdown in 1970 with no clear long-term recovery since. Averaged over the entire 1950 to 2010 period, rates of technological change η_{tech} are estimated to have been a respectable 3.5 % per year. But most of this growth took place in the first 20 years when it achieved 7.0 % per year. The latest 20 year period averaged just 1.4 % per year.

Improved access to energy reserves and raw materials explains most of the variability in η_{tech}. Coal power production expanded steadily at a rate of about 2 % per year. Oil reserves, on the other hand, expanded at an average 3.6 % per year between 1950 and 1970 but shrunk at an average 0.7 % per year between 1990 and 2010. The amount of energy required to access key raw materials such as cement, wood, copper and steel, dropped by an average 3.5 % per year between 1950 and 1970 implying rapid

efficiency gains. Since then, energy consumption and raw material consumption have grown at nearly equivalent rates.

Table 3 shows the 20 and 60 year averages of η and $\mathrm{d}\ln\eta/\mathrm{d}t$, and compares rates of technological change η_{tech} derived from either Eqs. (11) or (12). As a consequence of λ being a constant, calculated rates of return η are similar whether they are calculated from available energy statistics using Eq. (4) or from GWP statistics using Eq. (6). Both have averaged 1.6 % per year overall.

Innovation rates $\mathrm{d}\ln\eta/\mathrm{d}t$ have been positive overall, meaning rising rates of return. Still, they declined from 3.3 % per year between 1950 and 1970 to just 0.6 % per year between 1970 and 1990. The estimated average rate of technological change derived from Eq. (11) (i.e., $\eta_{\mathrm{tech}} = \mathrm{d}\ln\eta/\mathrm{d}t + 2\eta$) is 5.1 % per year, similar to what was derived for the 1950 to 1960 time period as discussed in Sect. 3.2. In comparison, the rate of technological change estimated from the physical parameters described in Table 2 averages 3.5 % per year, or about one third lower. Whether the residual is due to data uncertainties or theoretical considerations is unknown.

The hindcasts in Sect. 3.2 assumed a constant value for η_{tech} whereas the observed rates summarized in Table 3 point towards much higher variability. Perhaps the reasons a constant value works leads to high hindcast skill scores is because there is a timescale of decades for externally forced technological change to diffuse throughout the global economy (e.g., Rogers, 2010). Assuming a fixed value for η_{tech} represents this timescale by smoothing the impacts of the large 1950 to 1970 innovation impulse.

4 Positive skill in economic forecasts

Civilization has seen waves of logistic or sigmoidal growth throughout its history where an initial phase of exponential growth was followed by slower rates of expansion. Ancient Rome's empire increased to cover 3 500 000 km^2 in its first 300 years, but only a further 1 000 000 km^2 in its second; the Mongol empire extended to 20 000 000 km^2 within 50 years adding an additional 4 000 000 km^2 in the next (Marchetti and Ausubel, 2012). Growth at declining rates has also been noted in the adoption of new technologies (Rogers, 2010), the size of oil tankers (Smil, 2006), bacteria (Zwietering et al., 1990), and snowflakes (Pruppacher and Klett, 1997).

Viewed physically, these types of emergent behaviors are a response of a system to available reserves of potential energy and matter. Consumption of resources allows for expansion into more resources and then more consumption. The mathematical expression of the dynamics is fairly simple (Garrett, 2014), and it has been shown here how it can serve as a foundation for making 50 year hindcasts of the global economy (Fig. 7).

The accuracy of the hindcasts is due in part to a remarkable burst of technological change that occurred between 1950 and 1970. Figure 8 encapsulates the magnitude of the change. From available statistics, oil and gas reserves expanded faster than they were consumed. This changed around 1970. Reserves continued to be uncovered but they only barely kept pace with increasing demand. Early innovation and growth began to act as a drag on future innovation.

The mathematical form of this evolution is captured well by Eq. (11), at least assuming a fixed value for η_{tech}. Forecasting future scenarios may not be so easy. Civilization growth rates η have nearly completed their adjustment to the asymptotic value of $\eta_{\text{tech}}/2$ in Eq. (16). In other words, because innovation appears to have dropped to relatively low levels in recent decades, there is no longer a clear past signal that can be relied upon to propel civilization forward in a prognostic model; the post-war impulse has largely run its course.

This does not mean that the model described here lacks utility looking forward; rather, it implies that η_{tech} must be derived from something more than a fit to the past. To this end, three forces for technological change have been identified (Eq. 12). One is how fast civilization networks fray from such externalities as natural disasters. The others address the accessibility of raw materials and how fast new energy reserves are discovered relative to their rates of depletion. The challenge will be to provide

future scenarios for civilization that are based more explicitly on forecasts of resource discovery and natural decay.

5 Conclusions

This paper has taken the approach that economic systems are part of the physical universe, and that it should be possible to describe them accordingly. Prior work has found a constant relationship between global economic wealth and the total amount of energy consumption required to sustain civilization. From this point, two key economic rates were derived: a “rate or return” on wealth and energy consumption; and an “innovation rate” expressing how the rate of return evolves. The GWP growth rate is the sum of these two terms. Here it was shown that a simple prognostic economic model derived from physical first principles is able to make multi-decadal hindcasts of these economic variables with high positive skill.

In the 1950s and 1960s, civilization made exceptionally rapid gains in energy reserve discovery and resource extraction efficiency. This spurred a rapid increase in energy consumption and global wealth. More recently, the transition has been to more stable growth. Energy must be continually consumed to sustain existing economic circulations against dissipation and decay. Since civilization grew so rapidly in the middle of the last century, it seems that a large energetic burden was accumulated that restrains the pace of current expansion.

In Lewis Carroll’s *Through the Looking Glass*, Alice was urged by the Red Queen to run with her ever faster. But, “however fast they went, they never seemed to pass anything”. As the Red Queen put it “Now, *here,* you see, it takes all the running you can do, to keep in the same place.” If rates of technological change can be sustained at post-war levels of about 5 % per year, then theoretical considerations indicate that energy consumption and the GWP can be expected to continue grow at over 2 % per year. It appears, however, that technological change is slowing, in part because fossil fuel resource discovery has only just kept up with demand. In the future, should

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civilization growth ever stall due to a combination of diminishing returns and resource depletion (Bardi and Lavacchi, 2009; Murray and King, 2012), then simulations in Part 1 suggest that external forces may have the potential to push civilization into a phase of accelerating decline.

One such force is an increase in decay that might arise from natural disasters and environmental degradation (Arrow et al., 1995). Statistics presented here suggest that decay has been a comparatively weak player in the trajectory of recent civilization growth. This may change in the future if as expected atmospheric CO_2 concentrations reach "dangerous" levels and decay rates increase (Hansen et al., 2007; Matthews et al., 2009; Garrett, 2012a; Mora et al., 2013).

Here, a model has been introduced for making multi-decadal hindcasts of civilization evolution that allows for both positive and negative feedbacks to be represented in the coupled evolution of the human-climate system. Part 3 of this series will apply the model to forecast the evolution of civilization and the atmosphere in the century to come.

Appendix A: Innovation in traditional economic frameworks

The definition for innovation $\mathrm{d}\ln\eta/\mathrm{d}t$ that has been introduced here is very similar to definitions that have been made elsewhere. Traditional neo-classical growth models represent the nominal growth in "capital" K (units currency) as the difference between the portion s of production P that is a savings or investment, and capital depreciation at rate δ

$$\frac{\mathrm{d}K}{\mathrm{d}t} = (Y - W) - \delta K = sY - \delta K \quad \text{(A1)}$$

where individual and government consumption is represented by $W = (1-s)Y$. What is not saved or invested in the future is consumed in the present.

Labor L (units worker hours) uses accumulated investments in capital to enable further production Y according to some functional form $f(K,L)$. A commonly used repre-

sentation is the Cobb–Douglas production function

$$Y = AK^{\alpha}L^{1-\alpha} \quad \text{(A2)}$$

where, A is a "total factor productivity" that accounts for any residual in the ouput Y that is not explained by the inputs K and L. The exponent α is empirically determined from a fit to past data and $\alpha \neq 1$. This presents the drawback that the units for A are ill-defined and dependent on the scenario considered.

The Solow Growth Model Solow (1957) expresses the prognostic form for Eq. (A2) as

$$\frac{\mathrm{d}\ln Y}{\mathrm{d}t} = \frac{\mathrm{d}\ln A}{\mathrm{d}t} + \alpha\frac{\mathrm{d}\ln K}{\mathrm{d}t} + (1-\alpha)\frac{\mathrm{d}\ln L}{\mathrm{d}t} \quad \text{(A3)}$$

The term $\mathrm{d}\ln A/\mathrm{d}t$ has often been interpreted to represent technological progress. Such progress might be exogenous (Solow, 1957) or endogenous (Grossman and Helpman, 1990; Romer, 1994). If exogenous, then progress is considered to be due to an unknown external force. If endogenous, then it might come from targeted investments such as research and development.

An alternative approach that has been presented here is to consider civilization as a whole and subsume labor into total capital, in which case $\alpha = 1$ and $Y = AK$, where A has fixed units of inverse time. In this case, no fit is required, and the units are physical. Equation (A3) becomes equivalent to the expression $Y = \eta C$ in Eq. (5) where $\eta \equiv A$ and $C \equiv K$. The expression $\mathrm{d}\ln A/\mathrm{d}t$ that is assumed to describe technological progress in neo-classical frameworks (Eq. A3) is then mathematically equivalent to the definition for innovation $\mathrm{d}\ln\eta/\mathrm{d}t$ in the thermodynamic framework (Eq. 7).

Alternatively, in energy economics, the "production efficiency", or its inverse the "energy intensity", relates the amount of economic output that society is able to obtain per unit of energy it consumes (Sorrell, 2007). More efficient production is ascribed to technological change (e.g., Pielke Jr et al., 2008). The production efficiency can be defined mathematically as the ratio

$$f = Y/a \quad \text{(A4)}$$

From Eq. (A4) and the expression $a = \lambda C$ (Eq. 1) where λ is a constant, the production efficiency is linked to wealth C through

$$f = \frac{1}{\lambda}\frac{Y}{C} \tag{A5}$$

Rearranging, λf is equivalent to the rate of return η in Eq. (5) where it represents the conversion of economic wealth C to economic production Y through $Y = \eta C$. Wealth grows in proportion to the current production efficiency according to:

$$\eta = \frac{d\ln C}{dt} = \lambda f \tag{A6}$$

It follows that

$$\frac{d\ln \eta}{dt} \equiv \frac{d\ln f}{dt}$$

or that increasing the amount of economic production for a given energy input equates with innovation as it has been defined by Eq. (7).

As a side note, since η is also equal to the rate of growth in energy consumption (Eq. 4). This yields the counter-intuitive result that higher production efficiency accelerates growth in energy consumption. What is normally assumed is the reverse (Pacala and Socolow, 2004; Raupach et al., 2007). While the concept of "backfire" has been reached within more traditional economic contexts (Saunders, 2000; Alcott, 2005), it is a conclusion that remains highly disputed, at least where economies are viewed at purely sectoral levels (Sorrell, 2007, 2014).

Here, increased production efficiency $f = Y/a$ leads to an acceleration of energy consumption at rate $\eta = \lambda f$ because it expands civilization's boundaries with new and existing energy reservoirs (Garrett, 2014). Energy reservoirs may eventually be depleted, but in the instant efficiency yields the positive feedback that leads to ever faster rates of consumption.

A simple example is to contrast a sick child with a healthy child. Without having to know the "sectoral" level details of cellular function, it is clear that a healthy child will grow fastest. Health here is an implicit representation of the child's ability to efficiently convert current food consumption to growth and increased future consumption. Food contains the energy and matter that the child requires to grow to an adulthood. At this point, hopefully, a law of diminishing returns takes over so that weight is able to maintain a steady-state.

Appendix B: Estimated rates of technological change

Estimates of technological change rates η_{tech} require global scale statistics for the size of energy reserves, the rate of energy consumption, the rate of raw material consumption, and economic inflation. A challenge is that the reliability and availability of statistics diminishes the further back one goes in time. Accurate record keeping can be a challenge even for the most developed nations, much less for every nation. While global statistics for inflation might be available since 1970, they are given for only for a few countries in the 1950s (United Nations, 2010).

It is also not obvious how to sensibly represent raw material consumption. Cement, steel, copper and wood may be among the more obviously important components of the material flow to civilization, but their proportionate weights are far from clear. Steel is consumed in much greater volume than copper since it is a basic building material. But copper is an efficient conduit for electricity and equally important for civilization development.

With respect to energy reserves, the focus here is on fossil fuels since they remain the primary component of the global energy supply. Energy resources represent a total that may ultimately prove recoverable. Energy reserves represent the fraction of resources that is considered currently accessible given existing political and technological considerations. Unfortunately, there is no precise definition of what this means. Moreover, reserve and resource estimates are provided by countries and companies that may have political reasons to misrepresent the numbers (Höök et al., 2010; Sorrell et al., 2010).

The thermodynamic term ΔH_R in Eq. (14) represents the potential gradient that is available to drive civilization flows. It would seem to be most obviously represented by reserves rather than resources since reserves are what are most accessible and most directly exert an external pressure on civilization. A question that arises is how to provide some self-consistent way to add reserves of solid coal to reserves of natural gas and oil that diffuse to civilization as a fluid. Thermodynamically, any form of fossil fuel extraction requires some energy barrier to be be crossed, or an amount of work that must be done, in order to make the potential energy immediately available so that it can diffuse to the economy. The rate of diffusion is proportional to a pressure gradient (units energy density).

For example, well pressure forces a fluid fuel to the surface. Once the energy barrier of building the well is crossed, the magnitude of the pressure ca be related to the well reserve size ΔH_R (Höök et al., 2014). In contrast, coal reserves must be actively mined with a continuous energy expenditure. Even if the coal reserve is discovered, there remains a clear energetic cost in order to obtain an energetic return (Murphy and Hall, 2010; Kiefer, 2013). A hint at the importance of this energy barrier is that new fluid fuel reserves like oil and gas appear to affect economies much more rapidly than coal (Bernanke et al., 1997; Stijns, 2005; Höök et al., 2010, 2014), perhaps because they are more easily extracted and consumed.

In what is hopefully a defensible first step, the aforementioned concerns are addressed as follows for the purpose of calculating rates of technological change η_{tech}. Rates of growth of energy reserves (Eq. 14) are determined assuming that coal consumption is not reserve constrained, and rather that the closest solid equivalent to reserves of oil and gas in terms of accessibility is coal-fired power plants. Like discovering and exploiting an oil well, a power plant must be constructed, and it is only at this point that the coal reserve can be accessed to power civilization. Total reserves are

then the production weighted sum of the rates of growth of coal production capacity, oil reserves, and gas reserves (Rutledge, 2011; IHS, 2011).

Changes in civilization longevity are estimated using Eq. (13) for rates of decay and global inflation statistics since 1970 (United Nations, 2010). For the period before 1970, an average is taken of the respective inflation rates from the US, Great Britain, Japan, Germany, Italy and France (inflation.eu, 2014).

Rates of change in the specific energy of raw material extraction $e_s^{\text{tot}} = a/j_a$ (Eq. 15) are derived from statistics for global rates of energy consumption from all sources a (DOE, 2011), and from statistics for the consumption of iron and steel, copper, wood (excluding fuelwood) and cement (FAO, 2012; Boden et al., 2013; Kelly and Matos, 2014a, b). Wood and cement are treated as substitutable construction materials and are added according to their respective volumes. The total rate of change in j_a is then a simple average of the three rates of change: wood and cement; copper; and, iron and steel.

Statistics for the components of technological change are provided in Table 2.

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Table 1. For key economic parameters, a comparison between observed annual growth rates and 50 year predictions made assuming either a reference model of persistence or a hindcast model given by Eq. (11). Persistence is derived from historical rates between 1950 to 1960. The "observed" time period is 2000 to 2010. The skill score is derived from 1 – Error(hindcast)/Error(persistence) where error is derived relative to observed rates.

	Persistence (% yr^{-1})	Hindcast (% yr^{-1})	Observed (% yr^{-1})	Skill Score (%)
rate of return η ($d\ln a/dt$)	1.0	2.3	2.2 (2.4)	88 (96)
innovation rate $d\ln\eta/dt$	3.3	0.4	0.4	100
GWP growth rate $\eta + d\ln\eta/dt$	4.0	2.8	2.6	91

Table 2. Components of technological change expressed as 20 and 60 year averages of growth rates. See text and Appendix B for details.

Mean growth rates (% yr^{-1})	1950–1970	1970–1990	1990–2010	1950–2010
average raw materials per energy η_e	**3.5**	**–0.7**	**0.7**	**1.3**
cement and wood per energy	2.2	−0.8	−0.4	0.5
iron and steel per energy	4.6	−1.4	1.4	1.7
copper per energy	3.7	0.0	1.0	1.6
total fossil reserves η_R^{net}	**3.6**	**1.3**	**0.7**	**2.0**
oil reserves [production in EJ/year]	3.6 [59]	0.6 [133]	−0.7 [165]	1.1 [118]
gas reserves [production in EJ/year]	8.2 [22]	2.4 [62]	0.6 [98]	3.7 [60]
coal production [production in EJ/year]	2.2 [73]	1.9 [115]	2.3 [153]	2.2 [113]
change in longevity η_δ	**–0.1**	**0.2**	**0.2**	**0.2**
rate of technological change η_{tech}	**7.0**	**0.8**	**1.4**	**3.5**

Table 3. 20 and 60 year averages of rates of return (calculated using two independent techniques), innovation rates, and rates of technological change. Values are derived from Eqs. (11) and (12) and using data from Table 2.

Mean growth rates (% yr^{-1})	1950–1970	1970–1990	1990–2010	1950–2010
Observed rate of return $\eta = Y/\int_0^t Y\,dt'$ $(= (da/dt)/a)$	1.0	1.7 (1.6)	2.1 (2.0)	1.6
Observed innovation rate $d\ln\eta/dt$	3.3	1.6	0.6	1.9
Calculated technological change $\eta_{tech} = d\ln\eta/dt + 2\eta$	5.3	5.0	4.7	5.1
Observed technological change η_{tech}	7.1	0.8	1.2	3.5

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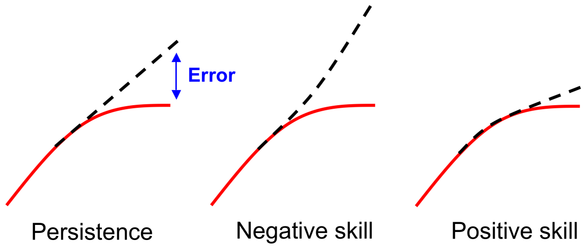

Figure 1. Positive skill in forecasts (black line) requires doing better than persistence in predicting future evolution of a quantity (red line).

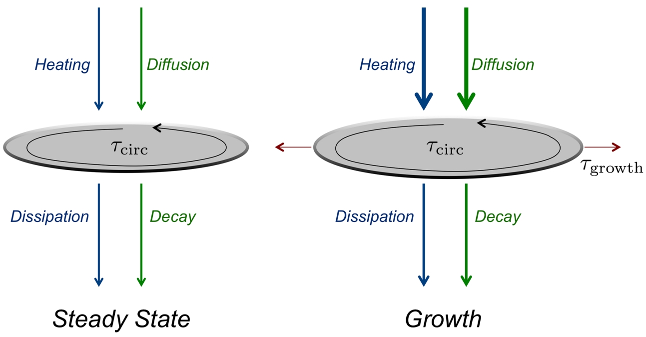

Figure 2. Thermodynamic representation of an open system. Reversible circulations within a system that lies along a constant potential have a characteristic time τ_{circ}. Circulations are sustained by a dissipation of a potential energy source that heats the system. The system maintains a steady state (left) because energetic (blue) and material (green) flows enter and leave the system at the same rate. Where there is a positive imbalance (right), the system grows irreversibly with time scale $\tau_{\text{growth}} \gg \tau_{\text{circ}}$.

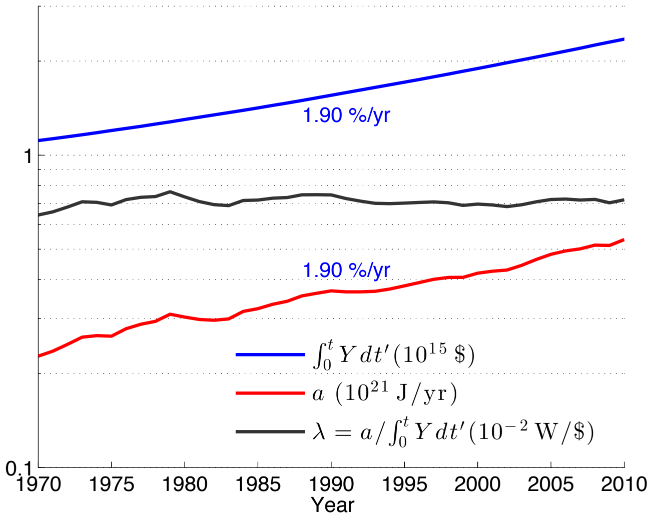

Figure 3. Rates of global energy consumption a, global wealth $C = \int_0^t Y(t')\,dt'$, and the ratio $\lambda = a/C$ since 1970. The average rates of growth η for a and C in percent per year are shown for comparison. The average value of λ is 7.1 ± 0.1 mW per year 2005 USD.

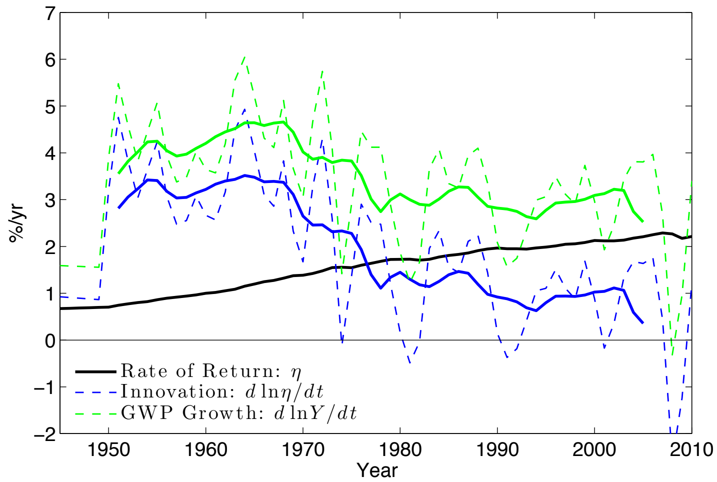

Figure 4. Time series of the rate of return, innovation and the GDP growth rate, evaluated at global scales and expressed in percent per year. Solid lines represent a running decadal mean (see Garrett, 2014 for methods)

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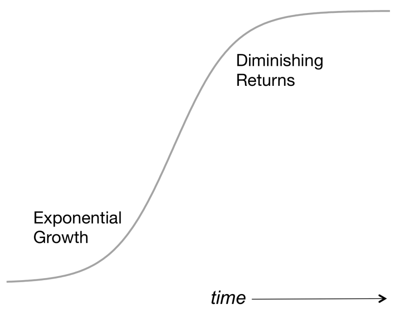

Figure 5. Illustration of the logistic curve

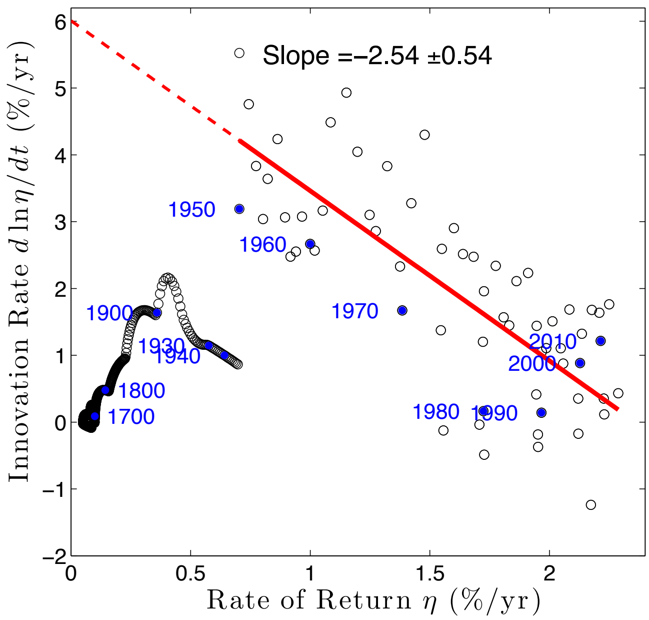

Figure 6. The global innovation rate $\mathrm{d}\ln\eta/\mathrm{d}t$ versus the global rate of return $\eta = Y/\int_0^t Y\mathrm{d}t'$ (Eq. 4). Select years are shown for reference. Since 1950, innovation is related to growth through the functional relationship $\mathrm{d}\ln\eta/\mathrm{d}t = S\eta + b$, where the slope and intercept shown by the red line, with 95 % confidence limits, are $S = -2.54 \pm 0.54$ and $b = 0.06 \pm 0.01$.

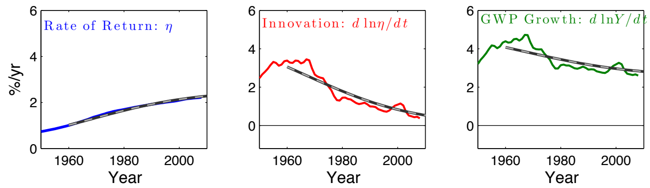

Figure 7. Gray lines: hindcasts starting in 1960 of the global rate of return $\eta = Y/\int_0^t Y\,dt'$, innovation rates $d\ln\eta/dt$ and the GWP growth rate $d\ln Y/dt = \eta + d\ln\eta/dt$. Hindcasts are derived from Eq. (16) assuming an average rate of technological change of 5.1 % yr^{-1} (dashed lines) derived from conditions observed in the 1950s. Solid lines: Observed decadal running means.

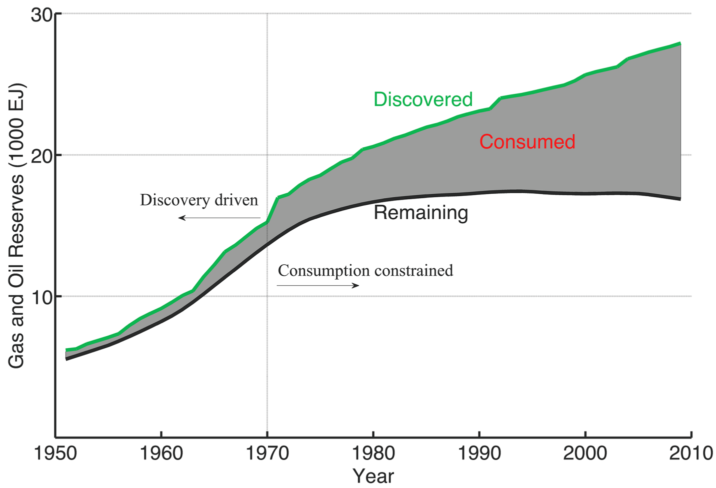

Figure 8. Discovered, consumed, and remaining global reserves of gas and oil since 1950 (source: IHS, 2011).

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