Dear Henk;

Please find the responses to the referees below in italics. The comments have helped to improve the paper, we hope that it is now ready for publication.

Sincerely,

Shaun

# Response to referees in italics:

Referee #1 (V.N. Livina)

The paper is well-written and generally well structured.

*Au: Thank you for your positive evaluation!* 

However, in pages 503-505 there are three subsections that have no counters (I presume these should be 2.3.2.1- 2.3.2.3). On the other hand, the text about forecast skill in section 2.5 may be separated into a subsection.

Au: we have added subdivisions, but it may depend of the policy of the journal, since this structuring is quite deep!

The abstract, in my opinion, should be re-formulated, to make it shorter and more concise. Terms like "enormous" and "huge" sound vague, whereas the term "stochastic memories" may be unclear to general readership.

Au: Thanks, we have changed the abstract to take into account the referee's concerns.

It would be interesting to see not only skills of the hindcasts, but also samples of time series compared.

Au: Yes, this hindcasts of individual series were the subject of a short GRL paper that was submitted a few weeks after the ESD paper. Although the initial referee comments were fairly positive, the paper was held up pending the result of the ESD paper! Therefore the answer to this question is not definitive, but we expect that the hindcast results on individual series (especially during the "pause" since 1998), to be published at about the same time as this ESD paper.

Note that in the paper by Livina et al (Physica A, 2013) "Forecasting the underlying potential governing the time series of a dynamical system", the scaling effects (long-term correlations) were taken into account in stochastic modelling, with dynamical forecast of probability density, rejection sampling for generation of a forecast time series, and reconstruction of correlations based on the previous part of the record. Similarly, this was validated in hindcasts on real climatic time series, up to 700 days (see the samples of time series in the paper). It would be really interesting to compare performance of the two approaches on the same time series in some kind of a joint exercise; however, as a reviewer I understand I may only recommend the paper as a reference for the revision.

Au: Yes, it would be very interesting to compare; we point this out in the new version. However the paper is already too long and this is work for the future!

### Further comments

The quality of some of the figures is not satisfactory. Multi-panel figures are combined without proper space adjustment. Figure 1 has unnecessary use of colour for labels, which are also placed in such a way as if they were typed over a ready graphic file. Fonts vary, some numbers are not readable. The same applies to Fig. 4. In addition, figure 4 has panel labels a-d in the caption but not in the panels themselves. In Figure 3, labels on x-axis are not readable; the axis can be shifted lower (to a value below y=0) for better readability; panels do not have labels a-c, which are used in the caption.

Au: We apologize, the original figures 1 and 4 were much better: at the stage of final submission we were asked to combine all the sub parts figures into single massive files and the quality was reduced as a consequence. This should not be an issue in the (final) ESD version.

In figure 8, the label on x-axis is missing. In figure 9, labels on the x-axes in the bottom row are missing. I also suggest reconsider the layout of 3- and 5- panel figures: they may look better if panels are stacked vertically (like matrix 3x1).

Au: OK, fixed. The production team can decide how to arrange them in the clearest way, we have improved the readability and labels. We have updated the figures as suggested.

Captions of the Figures 1 and 4 are excessively large; they contain comments that are more suitable for the main text discussion than for a figure caption.

Au: OK.

It is not clear to me why there are tables 1a and 1b: they may well be separate tables 1 and 2.

Au: We have renumbered all the tables.

Second order statistics is mentioned in pages 494 and 496 – with more explanation in the latter than in the former.

Au: The context and contents are different.

In page 498, the term "semi-Martingales" is given without explanation.

Au: It is a technical requirement to do with integration of stochastic processes: semimartingales are the most general stochastic processes with respect to which it is possible to integrate predictable processes in a reasonable way (i.e. the Itô and Stratanovich calculi). Here, we need only integrals of deterministic processes with respect to fGn (Wiener integrals), so we do not need to go into these issues.

In page 498, line 1, the word "usual" before "gamma function" is not necessary.

Au: Thanks.

Page 493, line 24: "see Fig.1a-e" does not need "below" (similarly in other places).

Au: Done.

After this reference to Fig.1, the next figure reference in page 496 is to Figure 4 rather than to Figure 2. I think the order of figures should be reconsidered according to their discussions in the text.

Au: I understand the reasoning, but fig. 4 cannot properly be discussed so early – it requires comments about the data set being used. On the other hand it is a shame not to indicate to the reader that there will later occur an empirical estimate of the exponents.

Table 2 is mentioned in the text before Table 1 (page 512).

Au: OK, all the tables have been renumbered!

'SD' is used first in page 513 without explanation of the abbreviation. *Au: Fixed.* 

In page 517, line 7: I think the equation should be 46 rather than 47.

Au: Yes, thanks.

In page 517, line 13: M\_{tt} has no comma between indices, whereas in other places it does.

Au: Yes, thanks.

In the caption of Fig.2, at the end of the text "(Sect.4 below)" makes no sense.

Au: Yes, thanks.

After displayed equations, before continuing inline text of the same sentence, commas are systematically missed – this issue is probably to be delegated to the publishing team.

Au: Yes, thanks.

## Response to Referee #2:

#### General comments

The paper by Lovejoy et al., entitled "The Scaling Linear Macroweather model (SLIM): using scaling to forecast global scale macroweather from months to decades", gives an important original contribution to long-term stochastic modeling of climate variables, taking into consideration the existence of memory in the climate system; the approach is validated in this paper for the temperature but it has the potential to be applied also for modeling other climate variables. The paper is well written, it offers a well structured reading and an adequate illustration of results. It gives a good review of fundamental background principles, namely of statistical physics, while also providing detailed explanations on the proposed approach and model. I therefore recommend its publication in the Journal. However, there are some issues about this manuscript that still require further attention from the authors, therefore revision is required, i.e. the manuscript should be subject to technical corrections. I ask the authors to take into consideration some of the comments listed below while revising their paper; the list of corrections is not exhaustive, but I will point out the main required actions which I hope are useful to the authors, to further improving the readability of the manuscript. Below, I refer to the web version of the paper, regarding page and line numbering.

## Specific comments

In my opinion, the relevancy of the analysis and modeling approach, in addition to the complexity inherent to this study area, justifies the somewhat lengthy paper, for full comprehension of the material. Results discussed in this manuscript are sufficient to support the interpretations and conclusions. However, I look forward to seeing soon publications reporting on further developments and applications of this work, that can complementary support the methodology proposed and allow for a better assessment of the potential of this model in climate forecasting, namely by confrontation of results with other approaches currently used.

#### **Technical corrections**

#### **Section 1:**

Page 491, line 24: please, clarify what is meant by "the same basic picture"; it could be linked to different previous sentences, in my view.

Au: OK, more information was added.

Page 492, line 18: please, be more specific on the meaning of "convenient physics notation";

Au: OK, clarified.

line 19: delete "usual"

Au: OK.

Page 492: make sure that all variables are defined in the text for eqs. 1, 2 and 3.

Au: Thanks, several were added.

Page 493, line 15: for clarity, substitute "OU" by "Ohrenstein-Uhlenbeck" *Au: OK.* 

#### Section 2:

Page 497, eq. 8: Please revise. Use format as elsewhere for the derivative and check that all variables are correctly in the equation.

Au: OK.

Page 498, line 1: in "usual gamma function", skip "usual". T(t) should be defined earlier in the text.

Au: OK.

Page 499, eq. 11: For consistency, please write the exponent H-1/2 as in e.g. eqs. 9 and 10: -(1/2-H). Define range of H values.

Au: OK.

Page 499, eq. 13: A new variable u is included. Please define.

Au: OK.

Page 501, eq. 21: Please revise/correct. The constant before the second integral should be deleted).

Au: OK.

Page 501, line 6: In the beginning of this line, the reference Mandelbrot and Van Ness (1968) seems to be misplaced;

Au: OK.

line 7: "can be eliminated"

Au: OK.

Page 502, line 9: for eq. 23, the interval for H', 0<H'<1, could also be added.

Au: OK.

Page 502, eq. 25: the interval for H, -1<H<0, could better be added.

Au: OK.

Parameter  $\lambda$  could be defined elsewhere (not included in the equation).

Au: OK, we added it to the text.

Page 503. Line 1: revise limits of H;

*Au: H<0 is enough here, but sentence improved.* 

line 8: please, clarify "the large  $\lambda$  formula"; eq. 27,

Au: OK.

better to give limits for H.

Au: Actually the formula is fine (valid for all H).

Page 504, 505: Section 2.3.2: the headings of subparts on Anomalies, Differences and Haar fluctuations should rather have font different from the headings of level 3, parent heading (maybe use italics, or different font size), to avoid confusion. *Au: OK.* 

Page 507, eq. 40: typo, delete the minus sign at the end of equation. *Au: OK.* 

Page 508, eq. 41: typo, delete the minus sign at the end of equation; define limits for  $\tau$ .

Au: OK.

Page 508, line 14: note sign error in equation.

Au: OK.

Page 510, line 5: maybe better refer to "innovations" instead of "record" *Au: OK.* 

Page 510, line 18: suggestion: use "ocean temperature" instead of "ocean data".

Au: OK.

Page 510, eq. 50 and page 511, eq. 51: please, check the arguments for  $F_H$ . *Au: OK.* 

#### Section 3

Check references to tables 1 and 2, not all are correct.

Au: OK.

Page 516, line 18: "... multiproxies (fig. 4c)."

Au: OK.

#### Section 4

Page 517, line 13: a comma is missing in subscript of M. *Au: OK.* 

Page 518, Section 4.2. The heading for this section is "results". This is most unexpected, especially because one can identify section 4, as a whole, to report results (although not explicitly stated in the heading of main section 4). Therefore, please, revise the heading of section 4.2.

Au: OK.

Page 520, line after eq. 56, please check text between brackets.

Au: OK.

## Figures:

In some figures, the authors opted by preparing longer than usual captions, but in my view these contribute to a more prompt understanding of the different figures, so I hope the journal sees no inconvenient and keeps the captions as proposed.

Au: Yes, we agree.

Axis titles: in some figures the axis labels are missing or cannot be read clearly. Please, revise and include units where applicable. In particular, check Figures 3, 8, 9. For consistency, use units K in Fig. 2.

Au: OK.

Figures composed by different panels: the panels are identified in the caption of the figures (e.g. a-e, Fig. 1) but not in the panels themselves. The readability of all such

figures should be guarantee by their adequate sizing during production of the final printed version of the paper.

Caption Fig. 1d, line 16: replace  $\tau$  by t in "... with forecast horizon = t = resolution ....", Caption Fig. 2, line 7 and 8: check respectively Table 2, not Table 1a; and delete "below".

Caption Fig. 3: line 1 - temperature "anomalies" could be indicated; line 7- add (c, bottom left); line 7 - in "The residues of the above", please clarify the meaning of "above".

Caption Fig. 4b, line 4: Clarify that the NASA GISS surface temperature series is anomaly data; line 16: correct "become".

Captions Fig. 5 to 8: there are some typos that need attention.

Au: All done:

### **Equations:**

Symbols, variables: Some symbols and variables used in several equations are not all defined in the text. Also, in some equations the range of pertinent variables are not always defined. This requires attention and revision.

Au: Thank you, many additional definitions and limts were added.

There is not consistency, throughout the text and particularly in equations, in relation to making explicit the dependency of some functions on given variables. Please, revise.

Reference to equations in the text: Please check that the correct equations are identified; in particular, from equation 37 onwards.

Some symbols are not in italic when in the text, please revise.

Au: thank you, all done.

#### **Other comments:**

Abbreviations in the text: Throughout the text, please check that all used abbreviations are defined/explained the first time they appear in the text. Gripenberg and Norros is a few places cited as Gripenberg and Norris. Please, correct.

Au: Thank you, fixed.

#### References:

Citation of references in the text: please, follow journal guidelines, there are several discrepancies. For example, line 12, page 491: (...) following Hasselmann (1976), (...).

Reference list: please, check references, which should be formatted respecting the journal guidelines. There are a few typos and discrepancies.

Au: the references have been improved.