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*Supplement of*

## **Uncertainty in temperature response of current consumption-based emissions estimates**

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## Supplementary material

### Analytical solution of consumption uncertainty for a one-sector, two-region

Here we present a simple analysis of the conditions under which a region's consumption uncertainty will be lower than its production uncertainty by virtue of uncertainty mixing through the MRIO model.

$$q_1 \quad q_2 = \begin{bmatrix} \frac{F_1}{x_1} & \frac{F_2}{x_2} & L_{11} & L_{12} & y_1 & 0 \\ & & L_{21} & L_{22} & 0 & y_2 \end{bmatrix}$$

$$q_1 = y_1 \frac{L_{11}}{x_1} F_1 + \frac{L_{21}}{x_2} F_2$$

$$\Delta q_1 = y_1 \frac{L_{11}}{x_1} \Delta F_1^2 + \frac{L_{21}}{x_2} \Delta F_2^2$$

$$a_1 = \frac{L_{11}}{x_1}, a_2 = \frac{L_{21}}{x_2}$$

$$\frac{\Delta q_1}{q_1} = \frac{a_1 \Delta F_1^2 + a_2 \Delta F_2^2}{a_1 F_1 + a_2 F_2}$$

$$\frac{\Delta q_1}{\Delta F_1} \frac{q_1}{F_1} = \frac{\frac{1}{\Delta F_1} (a_1 \Delta F_1^2 + a_2 \Delta F_2^2)}{\frac{1}{F_1} (a_1 F_1 + a_2 F_2)}$$

$$= \frac{a_1^2 + a_2 \frac{\Delta F_2^2}{\Delta F_1}}{a_1 + a_2 \frac{F_2}{F_1}}$$

$$= \frac{1 + \frac{a_2 \Delta F_2^2}{a_1 \Delta F_1}}{1 + \frac{a_2 F_2}{a_1 F_1}}$$

$$\frac{\Delta q_1}{\Delta F_1} \frac{q_1}{F_1}^2 = \frac{1 + \frac{a_2 \Delta F_2^2}{a_1 \Delta F_1}}{1 + \frac{a_2 F_2}{a_1 F_1} + 2 \frac{a_2 F_2}{a_1 F_1}}$$

$$r_1 = \frac{\Delta F_1}{F_1}, r_2 = \frac{\Delta F_2}{F_2}$$

$$\frac{\Delta q_1}{\Delta F_1} \frac{q_1}{F_1} = \frac{1 + \frac{r_2^2}{r_1} \frac{a_2 F_2}{a_1 F_1}}{1 + \frac{a_2 F_2}{a_1 F_1} + 2 \frac{a_2 F_2}{a_1 F_1} \frac{a_2 F_2}{a_1 F_1}}$$

$$= \frac{1 + \frac{r_2^2}{r_1} \frac{a_2 F_2}{a_1 F_1}}{1 + 1 + \frac{2}{\frac{a_2 F_2}{a_1 F_1}} \frac{a_2 F_2}{a_1 F_1}}$$

Therefore,

$$\frac{\Delta q_1}{\Delta F_1} \frac{q_1}{F_1} < 1$$

if and only if

$$\frac{\Delta F_2}{\Delta F_1} \frac{F_2}{F_1} < 1 + 2 \frac{L_{11} F_1 x_1}{L_{21} F_2 x_2}$$

$$\frac{\Delta F_2}{F_2} < \frac{\Delta F_1}{F_1} \frac{1 + 2 \frac{L_{11} F_1 x_1}{L_{21} F_2 x_2}}{1 + \frac{2}{\frac{a_2 F_2}{a_1 F_1}} \frac{a_2 F_2}{a_1 F_1}}$$

Since the radical term is always  $\geq 1$ , we can say that (at least) if  $\frac{\Delta F_2}{F_2} < \frac{\Delta F_1}{F_1}$  then  $\Delta q_1 \frac{q_1}{F_1} < \Delta F_1 \frac{F_1}{F_1}$ . In this simple case, we can simply say that region 1's production uncertainty is diluted by region 2's lower uncertainty to give a lower consumption uncertainty for region 1. This would be expected. However, we can also say from the analysis that there are conditions under which, even though the relative uncertainty of region 2 is larger than that of region 1, the consumption uncertainty of region 1 is still lowered by virtue of imports from region 2. Generalisation of this result to larger systems is left for future work.

**Table S1:** Sector aggregations, from GTAP sectors to 9 sector aggregation.

Agriculture	Paddy rice Wheat Cereal grains not elsewhere classified Vegetables, fruit, nuts Oil seeds Sugar cane, sugar beet Plant-based fibers Crops not elsewhere classified Cattle, sheep, goats, horses Animal products not elsewhere classified Raw milk Wool, silk-worm cocoons Forestry
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	Fishing
Mining	Coal Crude Oil Gas Minerals not elsewhere classified
Food	Meat: cattle, sheep, goats, horse Meat products not elsewhere classified Vegetable oils and fats Dairy products Processed rice Sugar Food products not elsewhere classified Beverages and tobacco products
Energy-intensive manufacturing	Paper products, publishing Refined petroleum Chemicals, rubber, plastic products Non-metallic minerals Ferrous metals Non-ferrous metals
Non energy-intensive manufacturing	Textiles Wearing apparel Leather products Wood products Metal products Motor vehicles and parts Transport equipment not elsewhere classified Electronic equipment Machinery and equipment Manufactures not elsewhere classified
Transport	Transport not elsewhere classified Sea transport Air transport
Services	Gas manufacture, distribution Water Construction Trade Communication Financial services not elsewhere classified Insurance Business services not elsewhere classified Recreation and other services Public Administration/Defence/Health/Education Dwellings
Electricity	Electricity
Households	Households