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# The dynamics of the Snowball Earth Hadley circulation for off-equatorial and seasonally-varying insolation

A. Voigt

Laboratoire de Météorologie Dynamique, IPSL, CNRS/UPMC – UMR8539, Paris, France

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Correspondence to: A. Voigt (aiko.voigt@lmd.jussieu.fr)

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## Abstract

I study the Hadley circulation of a completely ice-covered Snowball Earth through simulations with a comprehensive atmosphere general circulation model. Because the Snowball Earth atmosphere is an example of a dry atmosphere, these simulations 5 allow me to test to what extent dry theories and idealized models capture the dynamics of dry Hadley circulations. Perpetual off-equatorial as well as seasonally-varying insolation is used, extending a previous study for perpetual on-equatorial (equinox) insolation. Vertical diffusion of momentum, representing the momentum transport of dry convection, is fundamental to the momentum budgets of both the winter and summer 10 cells. In the zonal budget, it is the primary process balancing the Coriolis force. In the meridional budget, it mixes meridional momentum between the upper and the lower branch and thereby decelerates the circulation. Because of the latter, the circulation intensifies by a factor of three when vertical diffusion of momentum is suppressed. 15 For seasonally-varying insolation, the circulation undergoes rapid transitions from the weak summer into the strong winter regime. Consistent with previous studies in idealized models, these transitions result from a mean-flow feedback, because of which they are insensitive to the treatment of vertical diffusion of momentum. Overall, the results corroborate previous findings for perpetual on-equatorial insolation. They demonstrate 20 that an appropriate description of dry Hadley circulations, in particular their strength, needs to incorporate the vertical momentum transport by dry convection, a process that is neglected in most dry theories and idealized models. An improved estimate of the strength of the Snowball Earth Hadley circulation will also help to better constrain the climate of a possible Neoproterozoic Snowball Earth and its deglaciation threshold.

## 1 Introduction

25 Geological evidence suggests that during Neoproterozoic era (1000 to 541 million years before present), Earth might have repeatedly experienced global glaciations for

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many millions of years (Kirschvink, 1992; Hoffman et al., 1998; Hoffman and Schrag, 2000; Pierrehumbert et al., 2011). These glaciations are commonly referred to as Snowball Earth, a term inspired by the high albedo of such an Earth due to the hypothesized planetary-scale land glaciers and the completely sea-ice covered oceans.

5 The global ice cover entails surface temperatures well below freezing and, as a consequence of the Clausius–Clapeyron equation, an atmosphere essentially devoid of water. The Snowball Earth atmosphere thus belongs to the class of dry atmospheres, which traditionally have formed the starting point for theories of the large-scale atmospheric circulation. In particular, dry theories have been developed for the Hadley 10 circulation that dominates the zonal-mean circulation of Earth's tropics as well as other planets (Schneider, 2006).

Voigt et al. (2012) studied the dynamics of the Snowball Earth Hadley circulation using a comprehensive atmosphere general circulation model. Their primary motivation was to test to what extent assumptions commonly made in dry Hadley cell theories 15 are justified and to what extent dry Hadley cell theories are applicable to the Snowball Earth atmosphere. The use of a comprehensive atmosphere model that embodies sophisticated representations of diabatic processes such as radiation and small-scale turbulence made their study complementary to previous work with idealized atmosphere models that represent diabatic processes in a simplified manner. Voigt et al. 20 (2012) found that vertical diffusion of momentum, representing the dry turbulence and the momentum transport by small-scale (i.e. unresolved) eddies, is fundamental to the Snowball Earth Hadley circulation and in particular its strength. This result is important because dry Hadley cell theories and idealized models usually neglect vertical diffusion of momentum and assume that the flow is governed by its resolved large-scale 25 features. A potential limiting factor for the results of Voigt et al. (2012), however, is that the solar insolation maximum was located at the equator due to the use of perpetual equinox conditions. Because the location of the insolation maximum is known to affect the dynamics of the Hadley circulation (e.g. Lindzen and Hou, 1988; Walker and Schneider, 2005), I here expand the work of Voigt et al. (2012) to the cases of perpetual

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off-equatorial insolation maximum and seasonally-varying insolation. The primary goal of the present manuscript is to demonstrate that the results of Voigt et al. (2012) do not depend on the location of the solar insolation. Independent of the solar insolation profile, vertical diffusion of momentum is important to the momentum budgets of the 5 Hadley circulation and substantially weakens the circulation.

The simulations presented here with seasonally-varying insolation moreover connect to recent studies by Schneider and Bordoni (2008) and Bordoni and Schneider (2010) on rapid seasonal changes of the Hadley circulation in an idealized atmosphere 10 general circulation model coupled to a low thermal inertia surface. Schneider and Bordoni (2008) and Bordoni and Schneider (2010) described a mean-flow feedback that, supported by eddies, enables transitions in the dominant momentum balance from an 15 eddy-dominated summer and equinox regime to an angular-momentum conserving winter regime. In conjunction with the rapid changes in the Hadley cell strength and location, these circulations resembled the monsoons observed for today's Earth. The idealized atmosphere general circulation model used by Schneider and Bordoni (2008) and Bordoni and Schneider (2010), however, does not account for vertical diffusion of momentum, which raises the question to what extent vertical diffusion of momentum affects the rapid transitions.

Understanding the Hadley circulation of a Snowball Earth atmosphere is not only 20 important for the development of Hadley cell theories. A better characterization of the climate during Snowball Earth episodes also helps to constrain the deglaciation threshold of a hard Snowball Earth and to judge to what extent proposed scenarios for the survival of life during such harsh conditions appear plausible. The low heat 25 capacity of the ice surface allows large seasonal shifts of the Hadley circulation, yielding an indirect annual-mean circulation and, in contrast to Earth's modern climate, a zone of net evaporation around the equator (Abbot and Pierrehumbert, 2010; Pierrehumbert et al., 2011; Abbot et al., 2013). The equatorial net evaporation zone lowers the equatorial surface albedo because bare ice has a much lower albedo than snow (Brandt et al., 2005; Warren and Brandt, 2006; Dadic et al., 2013) and because of

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## 2 Model and simulation setup

I use the comprehensive atmosphere general circulation model ECHAM5 in idealized hard Snowball Earth conditions. The model and boundary conditions are the same as those used in Voigt et al. (2012), apart from differences in the solar insolation profile. In contrast to Voigt et al. (2012) who used perpetual equinox insolation with solar insolation maximum at the equator, I here apply perpetual insolation with an off-equatorial solar insolation maximum as well as seasonally-varying insolation (Fig. 1). The seasonally-varying insolation corresponds to a circular orbit with  $23.5^\circ$  obliquity. For the perpetual off-equatorial insolation profile, the insolation maximum is at  $23.5^\circ$  N. This profile does not represent the solsticial insolation conditions of the seasonally-varying insolation. Instead, it is chosen as one of many possible examples for off-equatorial insolation maxima and corresponds to the insolation at day 35 after vernal equinox of the seasonally-varying simulations. Interestingly, it will turn out below that the rapid Hadley circulation transitions in the seasonally-varying simulations are completed at day 35, after which the Hadley circulation strength remains almost constant for 130 days. Because of this, the Hadley circulation for the perpetual off-equatorial insolation profile closely resembles that for solsticial conditions. A 360 days calendar is used.

For completeness, I briefly summarize the idealized hard Snowball Earth conditions. A more thorough description is given in Voigt et al. (2012). A modern solar constant of  $1365 \text{ W m}^{-2}$  is used. Atmospheric  $\text{CO}_2$  is set to 300 ppmv;  $\text{CH}_4$ ,  $\text{N}_2\text{O}$ , CFCs, ozone, and aerosols are all set to zero. Sea-ice albedo is set to 0.75 everywhere. These choices yield tropical surface temperatures of around 220 K, and the simulated atmospheric conditions represent the very cold and essentially dry conditions during the early stage of a hard Snowball Earth glaciation. The low temperatures entail low atmospheric water vapor, which makes the simulated Snowball Earth atmosphere an example of a virtually dry atmosphere in which latent heating and moist convection have no appreciable effect on the circulation (Voigt et al., 2012).

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winter hemisphere, where a surface inversion forms<sup>1</sup>. The Hadley circulation consists of a strong winter cell and a much weaker summer cell. The center of the ascending region is located equatorward of the insolation maximum at 15° N. The winter cell attains its maximum of  $156 \times 10^9 \text{ kg s}^{-1}$  in the summer hemisphere at 3° N. As in the case of equinox insolation (Voigt et al., 2012), the winter cell extends up to 500 hPa and is embedded into a deep weak cell that reaches up to 200 hPa. The summer cell attains its maximum of  $11 \times 10^9 \text{ kg s}^{-1}$  at the maximum of the solar insolation at 23.5° N. The winter and summer cell separate at 20° N. While the winter cell is broad covering about 30° latitude, the summer cell is not only much weaker but also confined to a much narrower latitude band. Zonal winds are generally weak in the summer hemisphere, consistent with weak baroclinicity there. Stronger winds occur in the winter hemisphere, where winds are generally westerly and a subtropical jet develops at the poleward boundary of the winter cell. Upper-level easterlies of up to  $12 \text{ m s}^{-1}$  pertain close to the equator.

In steady state, the zonal-mean zonal momentum budget in pressure coordinates reads

$$\frac{\partial \bar{u}}{\partial t} = 0 = f \bar{v} + \bar{\zeta} \bar{v} - \bar{\omega} \frac{\partial \bar{u}}{\partial p} - \frac{1}{a \cos^2 \varphi} \frac{\partial (\bar{u}' \bar{v}' \cos^2 \varphi)}{\partial \varphi} - \frac{\partial \bar{u}' \bar{w}'}{\partial p} + D^u. \quad (1)$$

$D^u$  denotes the zonal-mean zonal momentum tendency due to vertical diffusion. Vertical diffusion represents the momentum transport by dry convection, which results from subgrid-scale dry eddies in response to unstable stratification and vertical wind shear. The notation is standard otherwise. In the Snowball Earth atmosphere, moist convection and horizontal diffusion do not substantially affect the zonal momentum budget, which is why they are not included in the above equation.

<sup>1</sup>Because the insolation maximum is located in the Northern Hemisphere, I will refer to this hemisphere as the summer hemisphere, and to the Southern Hemisphere as the winter hemisphere. I will use the same terminology for the Northern and Southern cells of the Hadley circulation.

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Vertical diffusion is of first-order importance to the Hadley circulation (Fig. 3). The summer cell is in viscous balance,  $f\bar{v} \simeq D^u$ . Vertical diffusion is also important for the winter cell, although the winter cell shows a more complex momentum budget. In the summer hemisphere, vertical diffusion is the primary process balancing the Coriolis force in the upper branch, with smaller contributions from the mean circulation. In the winter hemisphere, however, vertical diffusion is suppressed in the upper branch. Instead, meridional advection of mean relative vorticity dominates and the the cell is close to angular-momentum conservation,  $(f + \bar{\zeta})\bar{v} \simeq 0$ . Eddies are negligible for both the winter cell, as a result of easterly winds that shield the upper branch from the influence of the mid-latitude eddies of the winter hemisphere (Charney, 1969; Webster and Holton, 1982), and the summer cell, because baroclinicity and eddy activity are essentially zero there, consistent with the small meridional temperature gradients.

The dominant role of vertical diffusion on the strength of the Hadley circulation can be demonstrated by decomposing the strength of the winter and summer cells into contributions from the mean circulation, eddies, and vertical diffusion. Following Schneider and Bordoni (2008) and Bordoni and Schneider (2010), I do this rewriting the zonal momentum balance as

$$\bar{v} = \frac{1}{f} (M + E + D^u). \quad (2)$$

$M$  and  $E$  denote the momentum flux convergence due to the mean circulation and eddies, respectively. For a cell that attains its maximum  $\Psi$  at latitude  $\varphi_0$  and pressure  $p_0$ , the definition of the mass stream function,  $\Psi = -\frac{2\pi a \cos \varphi}{g} \int_0^{p_0} \bar{v}(\varphi_0, p) dp$  allows to decompose  $\Psi$  into a mean, an eddy, and a diffusive component,

$$\Psi = \Psi_M + \Psi_E + \Psi_D. \quad (3)$$

The mean component is defined as

$$\Psi_M = -\frac{2\pi a \cos \varphi}{f g} \int_0^{p_0} M(\varphi_0, p) dp; \quad (4)$$

analogous expressions hold for the other components. Table 2 shows that vertical diffusion of zonal momentum carries the entire strength of the summer cell and most of the winter cell.

Vertical diffusion not only impacts the zonal momentum budget but also the meridional momentum budget of the Hadley circulation. While the flow is in geostrophic balance throughout most of the atmosphere, the meridional momentum tendency due to vertical diffusion of meridional momentum approaches values comparable to those for the Coriolis force and the geopotential gradient in the region of the winter and summer cell. Therefore, and as in the case of equinox insolation, the meridional momentum budget is approximately given by

$$f \bar{u} \simeq -\frac{1}{a} \frac{\partial \bar{\Phi}}{\partial \varphi} + D^v, \quad (5)$$

where  $\Phi$  denotes the geopotential. Vertical diffusion of meridional momentum exhibits a distinct spatial pattern that coincides with the structure of the Hadley circulation (Fig. 4). This pattern implies that vertical diffusion mixes meridional momentum between the upper and lower branches of the circulation.

An analysis of the temperature tendencies in the region of maximum insolation reveals the reason for the strong vertical diffusion. While the atmosphere is approximately in radiative equilibrium above the tropopause at 500 hPa, the troposphere experiences radiative cooling due to the emission of longwave irradiance, which acts to destabilize the troposphere (Fig. 5). Because of the lack of moisture, latent heat release associated with moist convection is unable to provide the necessary stabilization. The same is true for vertical advection due to the dry-adiabatic temperature profile. Horizontal temperature transport by the Hadley circulation provides some of the stabilization in the upper troposphere by displacing isentropes from the vertical, which enables the dry Hadley circulation to transport energy (Voigt et al., 2012). Although this creates some upper-level heating, most of the stabilization needs to be carried by dry convection,

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which in the model is represented as vertical diffusion. This explains the strong vertical diffusion.

### 3.2 Impact of vertical diffusion of momentum on the strength of the Hadley circulation

5 The pivotal role of vertical diffusion in the momentum budgets of the Hadley circulation motivates to study how vertical diffusion of momentum impacts the strength of the circulation. To this end, I perform simulations in which vertical diffusion of momentum is suppressed in the upper branches of the Hadley circulation. To clarify whether vertical diffusion of zonal or meridional momentum is more important to the circulation 10 strength, I furthermore perform simulations with vertical diffusion suppressed only for either zonal or meridional momentum. Such an investigation is warranted by the expected opposing effects of vertical diffusion of zonal and meridional momentum (Voigt et al., 2012). The zonal momentum budget suggests that the circulation is driven by vertical diffusion of zonal momentum, based on which one expects that suppressing 15 vertical diffusion would weaken the circulation. In contrast, the meridional momentum budget suggests that vertical diffusion of momentum decelerates the circulation by mixing meridional momentum between the upper and lower branches. This suggests that suppressing vertical diffusion might lead to a stronger circulation.

To suppress vertical diffusion of momentum in the upper Hadley cell branches, I 20 follow the approach of Voigt et al. (2012) and set eddy viscosities to zero above 870 hPa. Vertical diffusion of dry static energy remains active throughout the atmosphere, i.e. eddy diffusivities remain untouched. While being readily implemented, the approach leads to an ambiguity in the calculation of turbulent kinetic energy (TKE), which is used to calculate eddy diffusivity and viscosity. To show that the qualitative 25 results are not sensitive to this ambiguity, I perform simulations in which TKE is calculated with either the full or the suppressed eddy viscosity (see Voigt et al., 2012, for details).

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## 4 Seasonally-varying insolation

This section studies the dynamics of the Snowball Earth Hadley circulation for seasonally-varying insolation. I repeat the simulations using a seasonal cycle in insolation as specified in Fig. 1 to determine to what extent the results for time-constant insolation carry over to time-varying insolation.

#### 4.1 Seasonal Hadley circulation, rapid transitions and associated momentum balances

During most of the year, the circulation is dominated by either the southern or the northern cell, depending on the sun's location (Fig. 6, top panel). From day 35 to 180 (counted from vernal equinox), when the sun is in the Northern Hemisphere, the southern cell is in the vigorous winter regime. The strength and location of the southern cell match those simulated for perpetual off-equatorial insolation: the stream function maximum of  $160 \text{ kg s}^{-1}$  is located in the Northern Hemisphere, and the cell extends from  $20^\circ \text{ N}$  to  $20^\circ \text{ S}$  (Fig. 6, middle panel). During the same period, the northern cell experiences the weak summer regime, with stream function values close to or equal to zero. Indeed, between day 80 to 160, the northern cell is collapsed. Reverse conditions occur 180 days later between day 215 to 360 when the sun is in the Southern Hemisphere.

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The Hadley circulation exhibits rapid transitions between states in which either the southern or the northern cell dominates. These transitions are so fast that the Hadley circulation in fact flips between these two states, yielding a square-wave dependence of the Hadley circulation strength on time. For example, at vernal equinox, the southern cell is in the weak summer regime, in which it remains until day 20. Thereafter, however, the southern cell transitions into the vigorous winter regime in only 15 days. At day 35, it has grown to its full strength and shifted its center into the Northern Hemisphere. The rapid transition is much faster than the change in solar insolation and manifests also in northward displacements of the vertical mass flux, the boundary between the southern and northern cell and the latitude of the maximum low-level potential temperature (Fig. 6, middle and bottom panels). In contrast, the collapse from the winter regime into the summer regime is slow. In terms of the cell strength, the collapse takes 75 days (from day 160 to 235), which is five times longer than the buildup. The different time-scales are explained by differences in the momentum budgets during the build-up and collapse as will be shown later in the section.

Because of the square-wave dependence of the circulation strength on time, many of the circulation characteristics found for perpetual off-equatorial insolation also apply to the solstitial circulation, which I here calculate as the July circulation from day 100 to 130. The tropopause in the summer hemisphere extends to about 500 hPa; potential temperatures are vertically uniform below (Fig. 7). The counterclockwise southern cell dominates the circulation, with easterlies prevailing in its upper branch. In the zonal momentum budget, the Coriolis force is primarily balanced by vertical diffusion of zonal momentum in the summer hemisphere (Fig. 8). In the winter hemisphere, the winter cell approximately conserves angular momentum, which is manifest in local Rossby numbers close to one<sup>2</sup> (Fig. 6, bottom panel). In the meridional momentum budget, vertical diffusion of meridional momentum mixes meridional momentum between the upper and lower branches, with magnitudes similar to the perpetual off-equatorial insolation (not shown). The solstitial circulation and the circulation for perpetual off-equatorial

<sup>2</sup>The local Rossby number is defined as  $Ro = -\bar{\zeta}/f$  (Schneider, 2006).

insolation differ in the high northern latitudes, but these differences are readily understood from differences in the insolation profile<sup>3</sup>. For example, the insolation maximum at the north pole in the solstitial conditions leads to a higher tropopause there and a weak counterclockwise cell, which is visible in the Coriolis term of the zonal momentum budget.

To decipher which processes are most important to the strength of Hadley circulation, I use the decomposition of the zonal momentum budget applied in Sect. 3. Because the Hadley circulation is dominated by the winter cell, I restrict the analysis to the southern cell and the period between days 0 and 235. During this period, the southern cell experiences the rapid transition into the vigorous winter regime, the fully-developed status as winter cell, and the collapse to the weak summer regime. Time-tendencies of the zonal wind above the winter cell maximum need to be taken into account, which is done by augmenting Eq. (3) by the inertial term,  $\Psi_I$ ,

$$\Psi_I = -\frac{2\pi a \cos \varphi}{f g} \int_0^{\rho_0} \frac{\partial \bar{u}}{\partial t}(\varphi_0, p) dp. \quad (6)$$

The resulting decomposition,

$$\Psi = \Psi_M + \Psi_E + \Psi_D + \Psi_I, \quad (7)$$

is calculated for consecutive pentads at the latitude of the center of the winter cell, which moves in time.

In the fully-developed phase of the winter regime, vertical diffusion is the most important contributor to the cell strength, carrying more than half of the cell strength. The mean circulation is the second-largest contributor. Eddies affect the cell strength only

<sup>3</sup>During solstice, the insolation exhibits maxima at 45° N and the north pole, with a fairly flat profile in between. The off-equatorial insolation used in Sect. 3 maximizes at 23.5° N and drops to lower values further poleward.

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marginally. The inertial term is substantial, but changes sign over the time of the existence of the fully-developed winter cell. Overall, apart from the inertial term, these findings match those for perpetual off-equatorial insolation (Table 2).

Before the transition into the winter regime, the cell is in linear transient balance, with the Coriolis force on meridional momentum being balanced by vertical diffusion of zonal momentum and the inertial term. During the transition, however, the balance is primarily between the Coriolis force and the momentum transport of the mean circulation, which indicates that the cell approximately conserves angular momentum. This is supported by calculations of the local Rossby number: during the transition, local Rossby numbers in the upper branch above the cell center are close to one, implying that the stream function and angular momentum contours coincide there (Fig. 6, bottom panel; local Rossby numbers are masked close to the equator because  $f$  is zero at  $\varphi = 0$ ) (Schneider, 2006).

Because the cell approximately conserves angular momentum during the transition into the winter regime, the cell responds directly to changes in the thermal driving (Schneider, 2006), and the mean-flow feedback described in Schneider and Bordoni (2008) explains the rapidity of the transition. Directly before the transition, at day 20, the maximum in low-level potential temperature and the boundary between the southern and northern cell are at the equator ( $\varphi_\theta = 0$ ;  $\varphi_\Psi = 0$ ). As the maximum solar heating moves farther poleward into the Northern Hemisphere,  $\varphi_\theta$  and  $\varphi_\Psi$  also move away from the equator into the summer hemisphere, which leads to a strengthening of the southern cell as expected from Lindzen and Hou (1988). On top of this, the advection of cold air in the lower branch of the intensifying southern cell pushes  $\varphi_\theta$  and  $\varphi_\Psi$  even further poleward, which leads to an additional strengthening of the circulation. Overall, the rapid transitions found here are consistent with the results of Schneider and Bordoni (2008) and Bordoni and Schneider (2010), showing that monsoon-like circulations are possible even in the absence of surface inhomogeneities provided the surface thermal inertia is sufficiently small.

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The dynamics the transition from the winter into the summer regime are qualitatively different from the dynamics during the transition into the winter regime. Momentum transport by the mean circulation is weak, leading to small local Rossby numbers in the upper branch of the cell (Fig. 6, bottom panel). Instead, vertical diffusion of zonal

5 momentum and the inertial term dominate the zonal momentum balance at the cell center, with opposing contributions. In particular, the inertial term prolongs the transition into the summer regime as its contribution to the cell strength is negative at the beginning but positive at the end of the transition. This explains why the transition into the summer regime is slow, in contrast to the the rapid transition into the winter regime for which the inertial term is small.

10 Vertical diffusion of meridional momentum is substantial throughout the winter regime and reaches the similar values as for the perpetual off-equatorial insolation. Vertical diffusion of meridional momentum is especially large during the transition into and out of the winter regime, when the maximum of solar surface heating is close to the center of the winter cell and vertical stability is low (not shown). For the same reason, vertical diffusion of meridional momentum is slightly smaller during the solstice period when the solar insolation maximum is at the pole, which leads to higher vertical stability at the cell center and lower eddy viscosities. Nevertheless, vertical diffusion of meridional momentum mixes meridional momentum between the upper and lower branches 15 of the winter cell at all times, consistent with its effect for perpetual off-equatorial insolation.

### 4.2 Impact of vertical diffusion on the strength of the Hadley circulation

In this subsection, I suppress vertical diffusion of momentum in the upper Hadley cell branch in simulations with seasonally varying insolation. I use the same approach as 20 for perpetual off-equatorial insolation in Sect. 3.2 and for equinox insolation.

The Hadley circulation intensifies by a factor of 2 to 3 when vertical diffusion of momentum is suppressed as is summarized for the winter cell in Table 3; values for the summer cell are not given as the latter collapses during solstice. The intensification

results from the suppression of vertical diffusion of meridional momentum, whereas suppressing vertical diffusion of zonal momentum only has a weak effect.

Figure 9 (bottom panel) shows the strength of the winter cell and its decomposition by means of the zonal momentum budget for the simulation U0V0T1. Results for the simulation U0V0T0 are similar. Vertical diffusion of momentum does neither affect the square-wave dependence of the winter cell strength on time nor the timing of the transitions. The latter likely is due to the fact that low-level eddy viscosities remain similar (Schneider and Bordoni, 2008; Bordoni and Schneider, 2010). When vertical diffusion of momentum is suppressed, the transition into the winter regime remains rapid with a time-scale of around 15 days, which is not surprising given that the transition into the winter regime is dominated by the mean circulation and vertical diffusion is small during the transition. Suppressing vertical diffusion of momentum leads to a slightly faster transition out of the winter regime compared to the simulation with active vertical diffusion (60 days compared to 75 days). To some extent this is because the winter cell is closer to the angular-momentum conserving limit when vertical diffusion of momentum is suppressed, so that the mean-flow feedback that causes the rapid transition into the winter regime can also operate during the transition out of the winter regime. However, the inertial terms causes the transition out of the winter regime to be markedly slower than the transition into the winter regime also when vertical diffusion of momentum is suppressed. Moreover, regarding the strength of the developed winter cell, suppressing vertical diffusion allows eddies to contribute substantially and even to dominate between day 130 to 160, in contrast to their only marginal contribution when vertical diffusion of momentum is active.

In summary, the results for seasonally-varying insolation corroborate those for perpetual off-equatorial and equinox insolation. They demonstrate that independent of the specified solar insolation, the net effect of vertical diffusion of momentum is to decelerate the Hadley cell due to mixing of meridional momentum between the upper and lower branches of the circulation.

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## 5 Impact of the Hadley circulation strength on the tropical hydrological cycle

From the perspective of atmospheric dynamics the Snowball Earth atmosphere is a dry atmosphere, with a hydrological cycle that is greatly suppressed with respect to Earth's modern climate. Even so, the Snowball Earth hydrological cycle impacts the flow of

5 sea-ice glaciers and determines the distribution of snow, both of which are important for the Snowball Earth climate and its deglaciation (cf. Sect. 1). In this section I therefore study how differences in the strength of the Hadley circulation caused by differences in the treatment of vertical diffusion affect the hydrological cycle.

The tropical annual-mean precipitation minus evaporation ( $P - E$ ) pattern exhibits 10 net evaporation in the region around the equator and net precipitation in the subtropics. This pattern, which is reversed to modern conditions, is consistent across models (Abbot et al., 2013) and results from the rapid seasonal transitions of the Hadley circulation, which imply that the ascent region in which precipitation outweighs evaporation is almost always located several degrees off the equator.

15 While the width of the equatorial net evaporation does not show any systematic variation with the strength of the Hadley circulation, the magnitude of the annual-mean  $P - E$  pattern scales linearly with the strength of the winter cell. The linear scaling also holds for simulations with perpetual off-equatorial insolation, which is expected because the annual-mean pattern results from the superposition of the two opposite 20 seasonal patterns. Thus, the linear scaling of the annual-mean pattern can be understood from the linear scaling of the seasonal pattern, which in turn can be understood from the moisture equation as is shown in the following.

Assuming steady state conditions, the vertical integral of the zonal-mean moisture 25 equation reads

$$\overline{P - E} = \int_0^{\rho_s} \nabla \cdot (\overline{q v}) dp = \frac{1}{a \cos \varphi} \int_0^{\rho_s} \frac{\partial \overline{q v} \cos \varphi}{\partial \varphi} dp. \quad (8)$$

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Overbars indicate a zonal mean,  $q$  denotes specific humidity,  $v$  the meridonal wind, and the notation is standard otherwise. The decrease of temperatures with height and the Clausius-Clapeyron equation entail that  $q$  is concentrated in the planetary boundary layer (BL). Thus, the vertical integral can be restricted to the planetary boundary layer, which leads to the following scaling of  $P - E$ :

$$\overline{P - E} \propto \nabla \cdot (\overline{q_{BL} v_{BL}}). \quad (9)$$

Here,  $q_{BL}$  and  $v_{BL}$  represent boundary layer values of  $v$  and  $q$ . Because the tropical circulation is dominated by its zonal-mean component, one can further write

$$\overline{P - E} \propto \nabla \cdot (\overline{q_{BL} v_{BL}}). \quad (10)$$

As the meridional mass flux in the boundary layer is proportional to the Hadley cell strength,  $H$ , we can relate  $\overline{P - E}$  to  $H$ ,

$$\overline{P - E} \propto \nabla \cdot (\overline{q_{BL} H}). \quad (11)$$

Finally, in the presented simulations changes in the vertical diffusion of momentum lead to strong changes in the Hadley circulation strength but do not substantially affect surface temperatures and boundary-layer humidity. Consequently, changes in  $P - E$  scale with changes in the Hadley cell,

$$\delta \overline{P - E} \propto \nabla \cdot (\overline{q_{BL} \delta H}). \quad (12)$$

This explains the linear scaling shown in Fig. 10.

## 6 Conclusions

In this paper, I study the dynamics of the Hadley circulation of a Snowball Earth atmosphere. To this end, I use a comprehensive atmospheric general circulation model in hard Snowball Earth boundary conditions, which leads to an essentially dry atmosphere. This offers a possibility to test the assumptions underlying dry theories of the Hadley circulation and to verify to what extent the results of dry Hadley circulations obtained with idealized atmospheric general circulation models hold up in a comprehensive model. The paper is motivated by and extends a previous study conducted for equinox insolation (Voigt et al., 2012). Here, perpetual off-equatorial and seasonally-varying insolation is used.

The main result of this paper is that vertical diffusion of momentum is fundamental to the dynamics of the Snowball Earth Hadley circulation. This is illustrated in the analysis of the zonal and meridional momentum budgets. Strong vertical diffusion is present in simulations with perpetual off-equatorial as well as seasonally-varying insolation. Together with the equinox result of Voigt et al. (2012), this demonstrates that the presence of strong vertical diffusion of momentum is independent of where the maximum solar heating is located.

The presence of strong vertical diffusion is explained by the radiative cooling of the troposphere. Because the radiative cooling of the troposphere is a generic atmospheric property, at least for atmospheres in which atmospheric shortwave absorption is not dominant (Pierrehumbert, 2010), it appears that strong vertical diffusion in the region of the Hadley circulation is unavoidable in dry atmospheres. This inference is supported by Snowball Earth simulations presented in Abbot et al. (2013), who used six comprehensive atmospheric general circulation models different from the one used here. In all of these models, and independent of the atmospheric CO<sub>2</sub> concentration and surface temperatures, local Rossby numbers were below 0.5 in many parts of the upper branch of the winter cell, in particular above its center. Because easterlies shield the center of the winter cell from mid-latitude eddies and stream lines are nearly horizontal

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there, the low Rossby numbers indicate the presence of substantial vertical diffusion of momentum. This suggests that differences in the strength of the winter cell between these models might, at least partly, explain by differences in the models' treatment of vertical diffusion of momentum. Nevertheless, studies with eddy-resolving models are 5 needed to better constrain the exact strength of vertical diffusion.

Vertical diffusion of momentum strongly impacts the strength of the Hadley circulation. Vertical diffusion weakens the circulation by mixing meridional momentum between its upper and lower branches. Consequently, neglecting vertical diffusion of momentum leads to a two to three times stronger circulation. An important implication of 10 this result is that theories for dry Hadley circulations ought to take into account vertical diffusion of momentum, although this might be difficult because of the small-scale nature of vertical diffusion. Moreover, the results suggest that vertical diffusion might alter the scaling laws for dry Hadley circulations derived in idealized atmospheric circulation models that neglect vertical diffusion of momentum (e.g., Held and Hou, 1980; Walker 15 and Schneider, 2006; Caballero et al., 2008). An improved quantitative understanding of the strength of the Snowball Earth Hadley circulation will also help to better constrain the climate of a hard Snowball Earth and its deglaciation threshold.

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**Table 1.** Summary of simulations with different treatment of vertical diffusion of momentum. All simulations are performed for perpetual off-equatorial as well as seasonally-varying insolation. The last column indicates whether full or suppressed eddy viscosities are used for the calculation of turbulent kinetic energy (TKE). See text for details.

Simulation	Vertical diffusion of		
	Zonal momentum	Meridional momentum	TKE
STD	on	on	on
U0V0T1	off	off	on
U0V0T0	off	off	off
U0V1T1	off	on	on
U0V1T0	off	on	off
U1V0T1	on	off	on
U1V0T0	on	off	off

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**Table 2.** Simulations with perpetual off-equatorial insolation and different treatment of vertical diffusion of momentum: strength of the winter and summer Hadley cell and decomposition into contributions from the mean circulation ( $\Psi_M$ ), eddies ( $\Psi_E$ ), and vertical diffusion ( $\Psi_{\text{vdiff}}$ ). The attribution is based on Eq. (3) and accurate to within 5 % of the cell strength as indicated by the residuum ( $\Psi_{\text{res}}$ ). All values in units of  $10^9 \text{ kg s}^{-1}$ .

Simulation	$\Psi$	$\Psi_M$	$\Psi_E$	$\Psi_D$	$\Psi_{\text{res}}$
Summer cell					
STD	11	0	0	11	0
U0V0T1	2	0	0	1	1
U0V0T0	2	0	0	2	0
U0V1T1	3	0	1	2	0
U0V1T0	3	0	1	2	0
U1V0T1	38	1	1	37	-1
U1V0T0	37	2	1	35	-1
Winter cell					
STD	156	27	7	117	5
U0V0T1	439	269	145	0	25
U0V0T0	288	133	141	0	14
U0V1T1	196	120	87	0	-11
U0V1T0	158	55	100	0	3
U1V0T1	431	98	8	320	5
U1V0T0	334	123	9	195	7

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**Table 3.** Strength of the winter cell for seasonally-varying insolation. Values represent the stream function maximum of the southern cell averaged between day 35 and 160 in units of  $10^9 \text{ kg s}^{-1}$ .

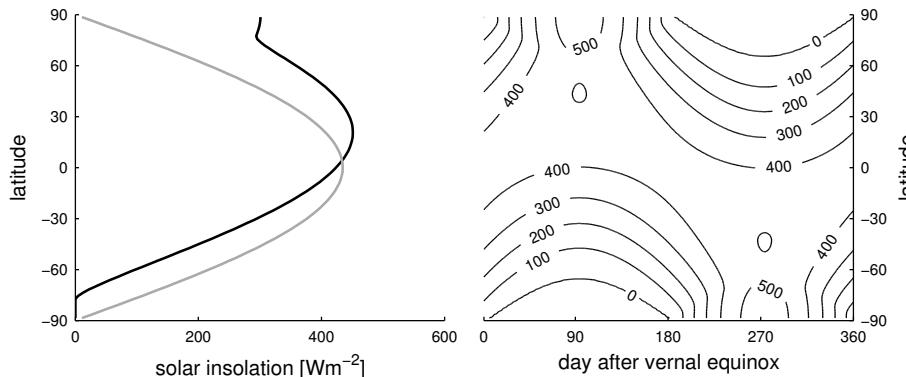
Simulation	Winter cell strength
STD	136
U0V0T1	375
U0V0T0	246
U0V1T1	178
U0V1T0	143
U1V0T1	380
U1V0T0	307

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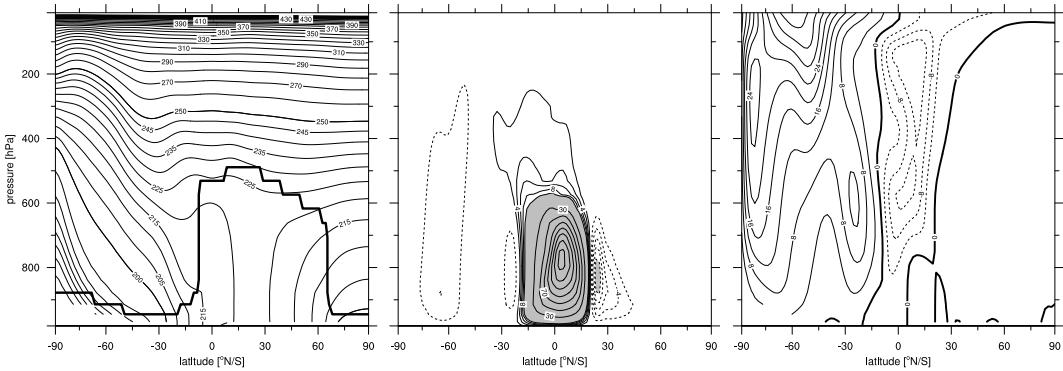


**Fig. 1.** Solar insolation at the top of atmosphere in  $\text{Wm}^{-2}$ . Left panel: perpetual off-equatorial insolation with maximum at  $23.5^\circ \text{ N}$ . For reference, the symmetric equinox profile used in Voigt et al. (2012) is shown in gray. Right: seasonally-varying insolation profile corresponding to a circular orbit with an obliquity of  $23.5^\circ$ . A 360-days calendar is applied, with days counted from vernal equinox.

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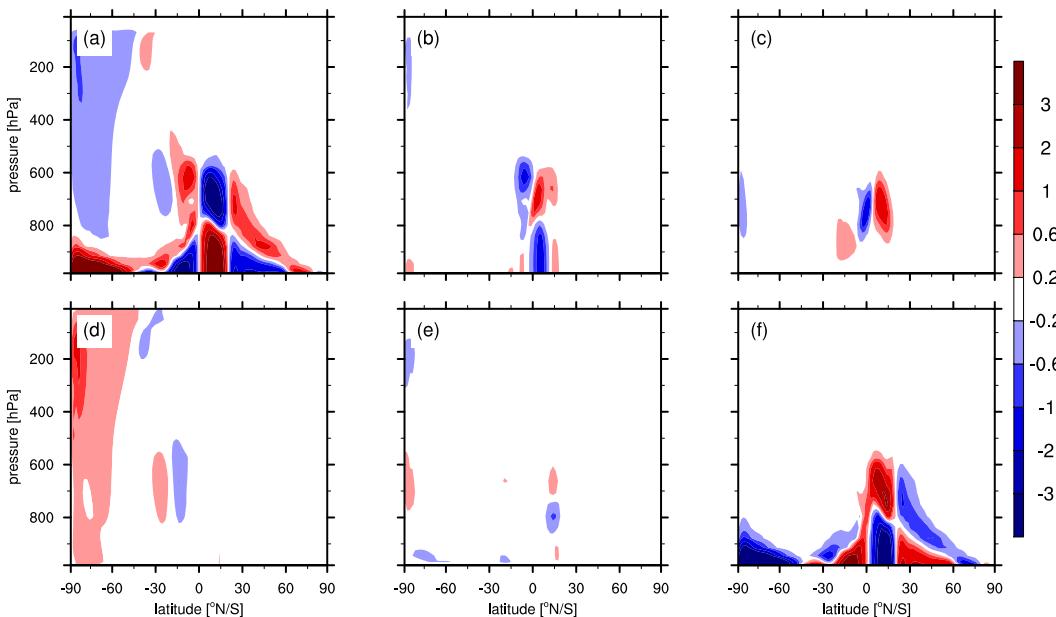


**Fig. 2.** Zonal-mean circulation of the Snowball Earth atmosphere for perpetual off-equatorial insolation. Left panel: potential temperature in K. Contour interval is 5 K between 200 and 250 K and 10 K otherwise. The black solid line depicts the tropopause (see text). Middle: mass stream function in units of  $10^9 \text{ kg s}^{-1}$ . Solid (dashed) contour lines represent counterclockwise (clockwise) circulation. Contour interval is 2 between  $-10$  and  $10$ , and 20 otherwise (gray shading). Left panel: zonal wind in  $\text{m s}^{-1}$ . Contour interval is 4, with a thick zero contour line.

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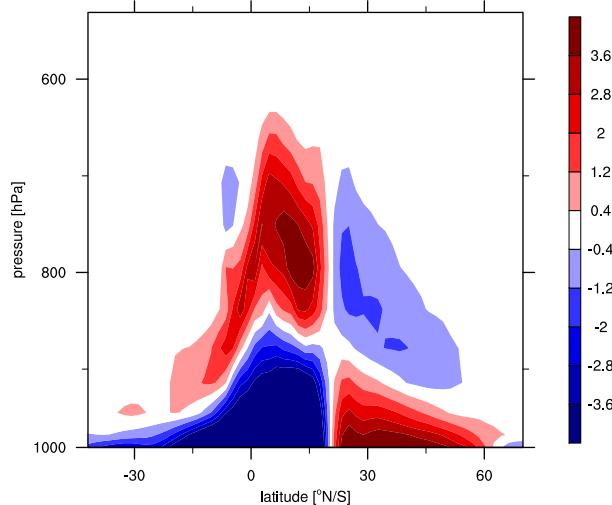
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**Fig. 3.** Zonal-mean zonal momentum budget for perpetual off-equatorial insolation in  $10^{-5} \text{ m s}^{-2}$ : **(a)** Coriolis force, **(b)** meridional advection of mean relative vorticity, **(c)** negative of vertical advection of mean zonal momentum, **(d)** horizontal contribution to eddy momentum flux convergence, **(e)** vertical contribution to eddy momentum flux convergence, and **(f)** vertical diffusion of zonal momentum. The contributions from moist convection (i.e. cumulus friction) and horizontal diffusion are negligible.

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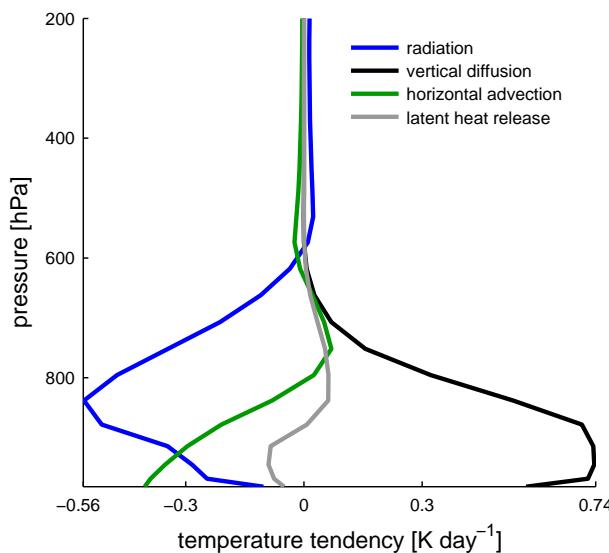


**Fig. 4.** Zonal-mean meridional momentum tendency due to vertical diffusion of meridional momentum in  $10^{-5} \text{ m s}^{-2}$  for perpetual off-equatorial insolation.

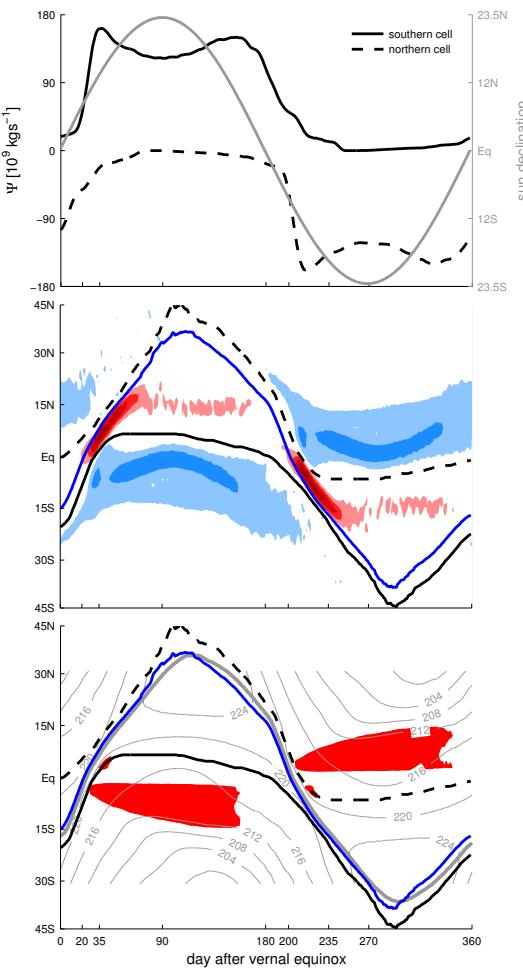
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**Fig. 5.** Temperature tendencies for perpetual off-equatorial insolation in the region of maximum solar heating, between 15 and 30° N. Vertical advection of temperature is negligible because temperatures are close to the dry adiabat.



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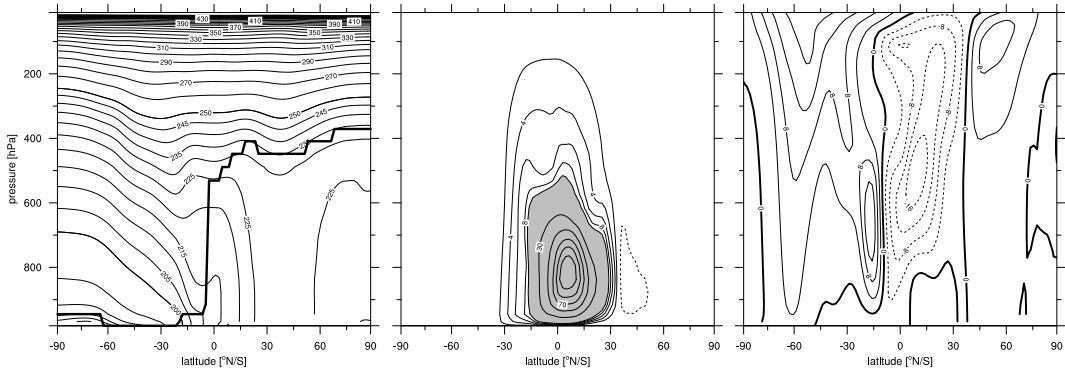
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**Fig. 6.** Hadley circulation characteristics for seasonally-varying insolation. Top panel: strength of the southern and northern cell in units of  $10^9 \text{ kg s}^{-1}$ . The sun declination in gray indicates the position of the sun. Middle: Vertical mass flux  $W = -\frac{2\pi a \cos(\varphi)}{g} \omega$  at 800 hPa in colored contours with contour interval of  $40 \times 10^4 \text{ kg m}^{-1} \text{ s}^{-1}$ ; upward mass flux in red, downward mass flux in blue. Bottom panel: potential temperature (gray contours) and its maximum (thick gray) at 900 hPa. Regions with local Rossby numbers above 0.7 at 700 hPa are shaded in red; note that regions in between  $2^\circ \text{ N/S}$  are masked as the Coriolis parameter is zero at the equator. The black and blue lines in the middle and bottom figures track the center of the northern (black dashed) and the southern cell (black solid) and the boundary between the two (blue solid).

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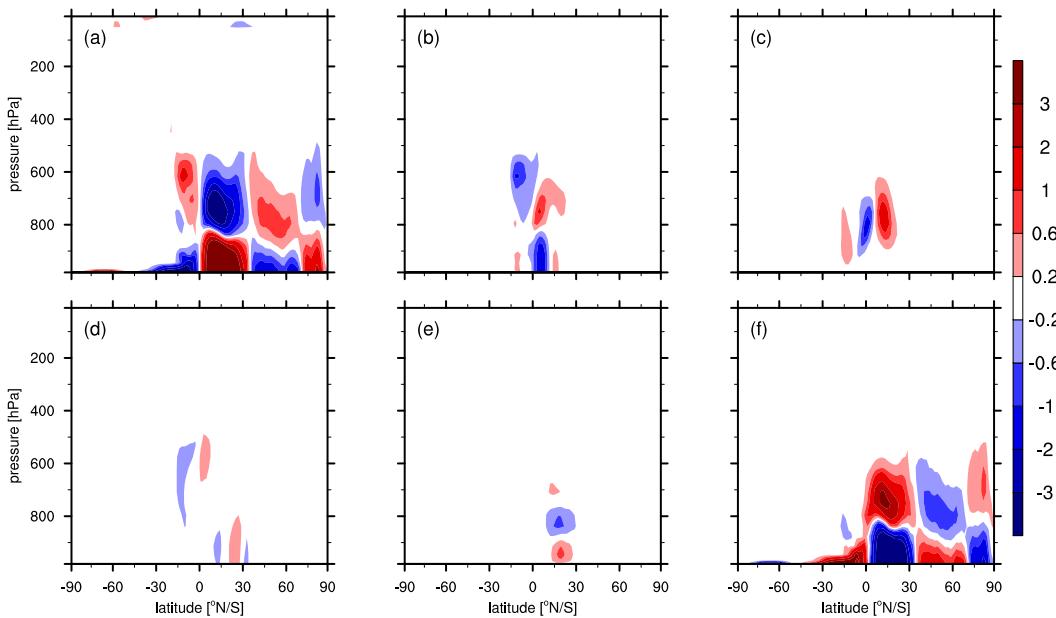


**Fig. 7.** Zonal-mean circulation for seasonally-varying insolation during solstitial conditions in July (day 100–130). Left: potential temperature in K. Contour interval is 5 K between 200 and 250 K and 10 K otherwise. The black solid line depicts the tropopause (see text). Middle: mass stream function in units of  $10^9 \text{ kg s}^{-1}$ . Solid (dashed) contour lines represent counterclockwise (clockwise) circulation. Contour interval is 2 between  $-10$  and  $10$ , and  $20$  otherwise (gray shading). Left: zonal wind in  $\text{ms}^{-1}$ . Contour interval is 4, with a thick zero contour line.

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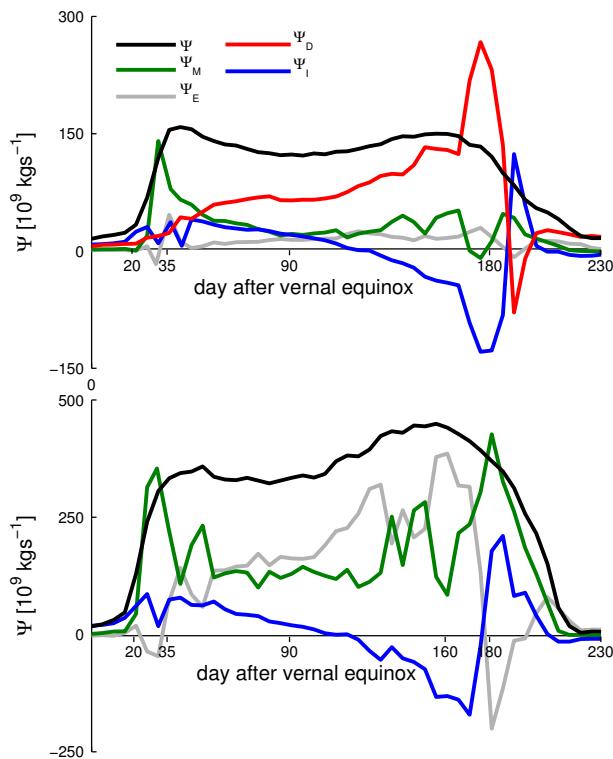


**Fig. 8.** Zonal-mean zonal momentum budget for seasonally-varying insolation during solstitial conditions in July (day 100–130) in  $10^{-5} \text{ m s}^{-2}$ : **(a)** Coriolis force, **(b)** meridional advection of mean relative vorticity, **(c)** negative of vertical advection of mean zonal momentum, **(d)** horizontal contribution to eddy momentum flux convergence, **(e)** vertical contribution to eddy momentum flux convergence, and **(f)** vertical diffusion of zonal momentum. The contributions from moist convection (i.e. cumulus friction) and horizontal diffusion are negligible. The zonal wind is in approximately steady-state in July, so that its time-tendency is small during this period.

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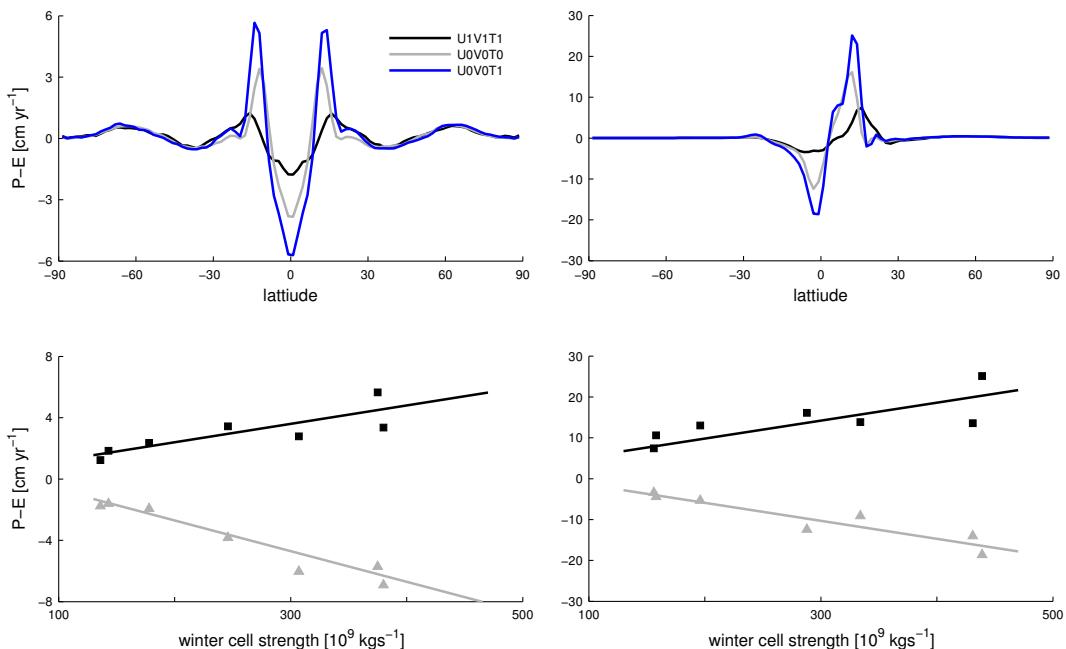
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**Fig. 9.** Strength of the winter cell ( $\Psi$ ) for seasonally-varying insolation as a function of time, and decomposition of  $\Psi$  into contributions from the mean circulation ( $\Psi_M$ ), eddies ( $\Psi_E$ ), vertical diffusion ( $\Psi_D$ ), and inertia ( $\Psi_I$ ). Top panel: standard simulation (U1V1T1). Bottom panel: simulation with suppressed vertical diffusion of zonal momentum, meridional momentum, and turbulent kinetic energy (U0V0T0).

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**Fig. 10.** Top panels: tropical pattern of precipitation minus evaporation ( $P - E$ ) for simulations with seasonally-varying insolation (left panels) and perpetual off-equatorial insolation (right panels) and different treatment of vertical diffusion of momentum. Bottom panels: peak values of the precipitation minus evaporation pattern versus strength of the winter cell. Triangles show the equatorial minimum, squares the maximum at around 10–15° latitude. The peak values scale linearly with the strength of the winter cell.

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