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Earth System
Dynamics
Discussions

Interactive comment on "Mathematical modelling of positive carbon-climate feedback: permafrost lake methane emission case" *by* I. A. Sudakov and S. A. Vakulenko

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We would like to thank referee #2 for the positive and constructive review. We will respond in detail to each of the comments below.

- However, they could better identify what is new by a direct comparison(s) to other approaches; the relevant discussion in section 5 has no references. The paper does not mention important previous work on modelling of thaw-lake growth and methane emission that can be found using a quick Google search.

We add the reviews of previous important works (e.g. Grigorjan, 1989; Khvorostyanov, 2008; Stepanenko, 2011) as well as we emphasize the feature of our approach in

compare with others.

-. . . Below I mention just a few (of many) points that should be clarified. The language is not precise and there are grammar errors.

We are really sorry for the great number of inaccuracy. It's the first experience of our joint work in interdisciplinary (mathematics and the earth science) field. We try to improve new version of this manuscript.

- There is no consistency in indicating the dependence of different variables on x,y,z, and t, to better distinguish variables from parameters.

We are formulated 3D Stefan problem where x,y,z are coordinates defined of a cylindrical domain. On the other hand temperature of phase transition in Stefan problem is detected in depends from change of coordinates and time. We are formulated Stefan problem as a standard problem of mathematical physics and also on Fig. 1 you can see geometry of this problem.

-On page 240, how can tanh(x/epsilon) be a solution to (9)? It does not depend on z.

Sorry, this is a misprint; tanh(z/epsilon) is correct.

- What is "the case C" mentioned on pages 241-242?

It's a error from early version of the manuscript.

- The same variable S is used in (15) and (17) in relation to different quantities. What is more, S is also used to denote the total lake area later in the paper. Above (16), "Delta s" should be "Delta S".

Thank you very much for this criticism of course we will correct it.

- Please explain how a differential equation for the lake-radius growth (18) can be a form of an algebraic equation (16).

Equation (16) is not algebraic, it is a complicated fully nonlinear parabolic equations

describing so-called motion of mean curvature. It is well studied. A number of publications concerned with it, including works of such famous mathematicians as P. L.Lions, and others. Some refs are given in the text. Among recent works where one can find further refs, Olivier Ley: Motion by mean curvature and level-set approach. Proceedings of a talk given at Muroran Institute of Technology (Japan), July 2004. Samuel Biton, Emmanuel Chasseigne et Olivier Ley : Uniqueness without growth condition for the mean curvature equation with radial initial data. Comm. Partial Differential Equations, 28 (9-10) : 1503-1526, 2003. (These works can be found on site of O. Ley and Internet). To see that it is PDE, let us represent the front as a level set in space defined by $V(x,t)=0$, where t is time. Then one has \frac{\partial V}{\partial t} - \Delta V + \frac{(D^2 V DV, DV)}{|DV|^2}=0. Eq. for R can be easily obtained if we assume that the level set is a sphere.

- On page 243 there are references to equations (20) and (21) which should clearly point to some other equations.

We think it is a misprint from LaTeX. It should be (7), (9).

- At the end of the first paragraph on page 244, the deterministic case is dR/dt = delta - kappa/R, where kappa = kappa(x,y,z,t) according to (16). This is in odds with equation (19) where R/dt = delta - mu/R, where mu is a constant. Why?

Unfortunately, we didn't understand this remark it is the same that concerning the question about equation (16) and (18) seems.

- Why is the Pareto law coefficient k referred to as feedback coefficient on page 249? I thought the feedback coefficient was denoted with gamma.

Sorry, this is misprint, we will correct it.

- What is different in figures 2-4: k or gamma? I cannot see any "true Armageddon" in figure 4. In fact, the temperature at 500 years is noticeably lower than in figure 3. Also, (rather unclear) captions claim that there is less methane in the presence of lake

influence? The lake area should have units of meter squared, not meter.

The main conclusion of this research that methane emissions from permafrost lakes will increase the global warming This situation was identified as "Arctic Armageddon " in article of Kerr,2010. We will correct this figures in advance informativity. Indeed we will correct this unit.

Best Regards, Ivan and Sergey

Interactive comment on Earth Syst. Dynam. Discuss., 3, 235, 2012.