Earth Syst. Dynam. Discuss., 3, 113–141, 2012
www.earth-syst-dynam-discuss.net/3/113/2012/
doi:10.5194/esdd-3-113-2012

Earth System Dynamics Discussions

On the relationship between metrics to compare greenhouse gases – the case of IGTP, GWP and SGTP

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Received: 18 January 2012 – Accepted: 2 February 2012 – Published: 6 February 2012

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Published by Copernicus Publications on behalf of the European Geosciences Union.

Abstract

Metrics for comparing greenhouse gases are analyzed, with a particular focus on the Integrated Temperature Change Potential (IGTP) following a call from IPCC to investigate this metric. It is shown that GWP and IGTP are asymptotically equal when the time horizon approaches infinity. The difference between IGTP and GWP is estimated for different greenhouse gases using an upwelling diffusion energy balance model with different assumptions on the climate sensitivity and the parameterization governing the rate of ocean heat uptake. It is found that GWP and IGTP differ by some 10 % for CH_4 (for a time horizon of less than 500 years), and the relative difference between GWP and IGTP is less for gases with a longer atmospheric life time. Further, it is found that the relative difference between IGTP and GWP increases with increasing rates of ocean heat uptake and increasing climate sensitivity. Finally, it is shown that IGTP is equivalent to the Sustained Global Temperature change Potential (SGTP) under standard assumptions when estimating GWPs, i.e. a constant background temperature and a constant background concentration of greenhouse gases.

1 Introduction

A range of metrics for comparing and aggregating the climate effect of different greenhouse gases has been proposed. When estimating GWP, the radiative forcing from a pulse emission, say one kg of gas X at time $t = 0$, is integrated until an arbitrary time horizon H, and divided by the result of an equivalent integration for the reference gas, usually CO_2. GWP is the standard option when comparing different greenhouse gases, e.g. in the Kyoto protocol. It was originally developed by Rodhe (1990), Shine et al. (1990) and Lashof and Ahuja (1990). See Forster et al. (2007), for IPCC AR4's estimates of GWP values.

An alternative to GWP that has received more attention recently is the Global Temperature change Potential (GTP). When estimating GTP the temperature response at

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time $t = H$ from a pulse emission of gas X at time $t = 0$ is divided by the equivalent temperature response for a reference gas, usually CO_2. It should be noted that GTP only considers the temperature response at $t = H$. Thus, it is different from GWP in the sense that GWP is an integrative measure (of the radiative forcing contribution over the entire period). GTP was initially proposed by Shine et al. (2005).

There is a renewed interest among researchers and policy makers to investigate the performance of other alternative metrics. A special meeting on metrics for comparing greenhouse gases was organized by the IPCC in Oslo in 2009 (IPCC, 2009). In its recommendations to the scientific community regarding research needs, IPCC (2009) writes that researchers should "develop new and refined metrics", e.g. a metric based on "the integral of the temperature change" following a pulse emission of a gas X compared to "the integral of the temperature change" following a pulse emission of CO_2.

Such a measure has been discussed previously by for instance Fisher et al. (1990), Rotmans and den Elzen (1992), Shine et al. (2005, p.298), IPCC (2009), Gillet and Matthews (2010) and Peters et al (2011). We here refer to this as metric as the **I**ntegrated **G**lobal **T**emperature change **P**otential (IGTP). Note that it is not GTP that is integrated, but the temperature response from gas X divided by the integrated temperature response from the reference gas.

In addition to GWP, GTP and IGTP, Shine et al (2005) proposed a **S**ustained **G**lobal **T**emperature change **P**otential (SGTP) as the temperature response at time $t = H$ of a gas X emitted with a constant (sustained) rate ($1\,\mathrm{kg\,yr^{-1}}$) divided by the temperature response at time $t = H$ following sustained emissions of CO_2 ($1\,\mathrm{kg\,yr^{-1}}$). Fisher et al. (1990) also estimated halocarbon global warming potentials, i.e. a metric for the integrated warming effect of halocarbons, that was defined in the same way as SGTP with the main difference that infinite time horizons were used. They could use infinite time horizons since halocarbons have exponential decay rates (which ensure convergence of the relevant integrals).

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The purpose of our paper is to analyze the properties of IGTP and demonstrate its relationship with GWP and SGTP. Relations between IGTP, SGTP and GWP have been noted in the literature. O'Neill (2000) showed that IGTP and GWP are equivalent when the time horizon approaches infinity under special conditions. Shine et al find a "near equivalence" between SGTP and GWP (and thus between IGTP and GWP as IGTP and SGTP, as will be shown in this paper, are identical under "standard" assumptions when calculating GWP values). Further, Fisher et al. (1990) observed numerically that steady state values for SGTP (or HGWP in their terminology) are closely related to IGTP. However, these observations have received little attention and this partly explains why IPCC warranted further investigations of IGTP.

In a very recent paper, just published as we typed the final words into this manuscript, Peters et al observe a close relationship between IGTP and GWP. In their paper, they conclude that "further modeling would be required to confirm these observations". In our paper, we use an upwelling diffusion energy balance model instead of the impulse response approach they take to estimate IGTP. In addition, we carry out a more systematic sensitivity analysis with respect to the climate sensitivity and vertical heat diffusivity.

In Sect. 2, we present our model. The results are presented in Sect. 3 which also contains our sensitivity analysis. In Sect. 4 we explain the results. Conclusions are given in Sect. 5. In the appendices, we formally show that IGTP and SGTP are equivalent measures (under standard assumptions about linearity in atmospheric adjustment times and constant radiative efficiencies), and that IGTP and GWP are asymptotically equal as the time horizon approaches infinity.

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2 Method

In this section, we present the method used to calculate IGTP (and GWP). IGTP is defined as

$$\mathrm{IGTP}(H) = \int_0^H T_{PX}(t)\,\mathrm{d}t \Big/ \int_0^H T_{P,CO_2}(t)\,\mathrm{d}t \tag{1}$$

where $T_{PX}(t)$ is the temperature response at time t from a pulse emission of 1 kg of gas X at time 0 (and similarly for the temperature response for a pulse emission of 1 kg of CO_2). In Appendix A we show that IGTP is identical to SGTP (under standard assumptions when calculating GWPs). Thus all results presented in this paper that holds for the relationship between IGTP and GWP also hold for the relationship between SGTP and GWP.

Further, GWP is defined as

$$\mathrm{GWP}_X = \frac{\int_0^H C_{PX}(t)\,F_X\,\mathrm{d}t}{\int_0^H C_{P,CO_2}(t)\,F_{CO_2}\,\mathrm{d}t} \tag{2}$$

where $C_{PX}(t)$ is the mass of greenhouse gas X in the atmosphere at time t following a pulse emission (of 1 kg) at time $t = 0$, and F_X is the radiative efficiency per kg of gas X in the atmosphere. We assume that the background concentration of greenhouse gases (and thus the radiative efficiency) is constant, which is standard when estimating GWP.

In order to estimate IGTP numerically, we use an Upwelling Diffusion Energy Balance Model (UDEBM). The model is parameterized to a give a response in line with more complex climate models (see Johansson, 2011). In the UDEBM the surface of the globe is divided into one fraction for the ocean (and the troposphere above it) and

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one fraction for the land surface (and troposphere above it). The equilibrium temperature change is set to be 30 % higher for the surface air over land than over the oceans (see Raper et al., 2001 and Meinshausen et al., 2011). The marine surface air temperature increase is assumed to be 30 % higher than the ocean surface temperature increase due to retreating sea ice cover (see Raper et al., 2001 and Meinshausen et al., 2011). The warming of the water that downwells in the polar regions is assumed to be increased by a fifth of the increase in the global average sea surface temperature. The heat capacity of the land fraction is set to zero. All these model assumptions are basically standard in UDEBMs and similar models (see Raper et al., 2001; Shine et al., 2005; Meinshausen et al., 2011; Baker and Roe, 2009; Olivié and Stuber, 2010).

In the standard set-up, climate sensitivity is set to 3 K for a doubling of the atmospheric CO_2 concentration in line with IPCC's best estimate (Solomon et al., 2007). The likely range for the climate sensitivity is according to IPCC 2–4.5 K for a doubling of the atmospheric CO_2 concentration. We use this range in our sensitivity analysis. The ocean heat mixing in the UDEBM is determined by the vertical heat diffusivity and the upwelling rate. The upwelling rate is set to 4 m yr^{-1} (see Raper et al., 2001; Johansson, 2011; Meinshausen et al., 2011) and the base case diffusivity is set to 2 cm^2 s^{-1}. In order to emulate the ocean heat uptake and the surface temperature response in more complex models a diffusivity on the range between 0.5 and 5 is often used in UDEBMs (see Raper et al., 2001; Johansson, 2011; Meinshausen et al., 2011; Baker and Roe, 2009; Olivié and Stuber, 2010). In the sensitivity analysis we set this parameter at 0.5 and 4 cm^2 s^{-1} as alternatives to our base case assumption. In Appendix B, we demonstrate that our choice for these parameter values, in the base case as well as in the sensitivity analyses, are compatible with the measured global average surface temperature change over the past hundred years.

For the atmospheric adjustments times for CH_4, N_2O, SF_6 and CO_2, we use the assumptions that are used when estimated GWP in IPCC AR4 (see Forster et al., 2007). The radiative efficiency measured per kg gas is also estimated from Forster et al. (2007).

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In addition we assume that the climate sensitivity of gas X and CO_2 are the same. In many cases, in particular for forcings that are not globally homogenous, they are not (see e.g. Hansen et al., 2005), but it could be argued that under such conditions global warming potentials should be recalculated so that the climate efficacy of gas X is taken into account. In our calculations, we have assumed that the climate efficacies are equal to one throughout the paper.

3 Results: Comparison of IGTP and GWP

We start by presenting numerical estimates for the GWP and IGTP for CH_4, N_2O and SF_6 in Table 1.

It can be noted that the GWP and IGTP values are close (see also Azar and Johansson, 2011 and Peters et al., 2011). One may also note that the IGTP values for methane are slightly higher than its corresponding GWP values, whereas the opposite holds for SF_6. We will return to this feature later on in the paper and see how it results from the fact that the gases have significantly shorter and longer perturbation life times than CO_2.

The estimates for the IGTP/GWP ratios as a function of the time horizon, H, for CH_4, N_2O and SF_6 with CO_2 as the reference gas, can be seen in Fig. 1. The difference between GWP and IGTP is typically around or less than around 10 %.

Sensitivity analysis

In this subsection, we focus on how the ratio between IGTP and GWP is affected by changes in the vertical heat diffusivity and climate sensitivity.

An increase in the vertical heat diffusivity result in that the temperature increases more slowly in response to changes in the radiative forcing (see for example Hansen et al., 1985 and Johansson, 2011). This magnifies the difference between the equilibrium temperature change for a given forcing and the actual temperature change. As a

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consequence, the higher the heat diffusivity, the higher the inertia of the climate system, and the more IGTP will deviate in relative terms from GWP for all greenhouse gases (see Fig. 2). Decreasing the vertical heat diffusivity has the opposite effect, i.e. the IGTP/GWP ratio will for both short lived and long lived greenhouse become closer to unity.

Changing the climate sensitivity has a similar effect as changing the effective vertical heat diffusivity, since a larger climate sensitivity implies that the temperature responds, in relative terms, more slowly to changes in radiative forcing, (see e.g. Hansen et al., 1985). Hence, increasing the climate sensitivity magnifies the difference between the equilibrium temperature change for a given forcing and the actual temperature change. This is shown for CH_4, N_2O and SF_6 in Fig. 3. Decreasing the climate sensitivity has the opposite effect, i.e. the IGTP/GWP ratio will for both short-lived and long-lived greenhouse become closer to unity.

Finally, in Appendix C, we present numerical values for IGTP for CH_4, N_2O and SF_6 for different assumptions on the vertical heat diffusivity and the climate sensitivity. In most cases, IGTP values change modestly for even large changes in these two parameters. For example, a change in the time horizon (from 20 to 500 years) changes IGTP values by a factor of two for SF_6 and N_2O, and almost a factor of 10 for methane, but changes in the climate sensitivity and the heat uptake only affect the IGTP values by a few percent (in the absolute majority of cases).

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4 Interpreting the relationship between GWP and IGTP

In this section we aim to explain and interpret the results presented in Sect. 3. The ratio of IGTP and GWP is given by dividing Eq. (1) with Eq. (2), i.e.

$$\frac{\mathrm{IGTP}(H)}{\mathrm{GWP}(H)} = \frac{\int_0^H T_{\mathrm{P}X}(t)\,\mathrm{d}t}{\int_0^H T_{\mathrm{P},\mathrm{CO}_2}(t)\,\mathrm{d}t} \Bigg/ \frac{\int_0^H C_{\mathrm{P}X}(t)\,F_X\,\mathrm{d}t}{\int_0^H C_{\mathrm{P},\mathrm{CO}_2}(t)\,F_{\mathrm{CO}_2}\,\mathrm{d}t}. \quad (3)$$

The near equivalence between IGTP and GWP can be understood by rewriting the ratio IGTP/GWP in the following way

$$\frac{\mathrm{IGTP}(H)}{\mathrm{GWP}(H)} = \frac{\int_0^H T_{\mathrm{P}X}(t)\,\mathrm{d}t}{\int_0^H \lambda\,C_{\mathrm{P}X}(t)\,F_X\,\mathrm{d}t} \frac{\int_0^H \lambda\,C_{\mathrm{P},\mathrm{CO}_2}(t)\,F_{\mathrm{CO}_2}\,\mathrm{d}t}{\int_0^H T_{\mathrm{P},\mathrm{CO}_2}(t)\,\mathrm{d}t}. \quad (4)$$

Here the first ratio on the right hand side is the integrated (transient) temperature change divided by the integrated equilibrium temperature change for a gas X (following a pulse emissions). The equilibrium and the transient temperature responses of CH_4 are illustrated in Fig. 4 (upper panel). As seen in Fig. 4 the equilibrium temperature response is higher than the transient response the first two decades due to heat uptake by the oceans, thereafter the transient temperature response is higher than the equilibrium response due to heat release from the oceans. The integrated equilibrium temperature response (the area under the blue curve) is larger than the integrated transient temperature response (the area under the red curve) for any time horizon H, but the areas under the two curves approach the same value asymptotically as H approaches infinity.

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This asymptotic behavior can be understood in physical terms; the (integrated) radiative forcing must eventually manifest itself in (integrated) temperature change (see Appendix D for a formal proof of this). Thus, the first ratio on the right hand side in Eq. (4) will be less than one and asymptotically approach one when the time horizon H approaches infinity. Similar arguments hold for the second ratio, as will be discussed below, and for that reason the IGTP to GWP ratio becomes equal to unity as the time horizon approaches infinity, see Appendix D (O'Neill, 2000 and Peters et al., 2011).

The second ratio on the right hand side of Eq. (4) is the integrated equilibrium temperature change divided by the integrated temperature change for a pulse emission of CO_2 (see lower panel, Fig. 4). Here the same arguments can be used as for CH_4 above, although the time scales involved when approaching unity is much longer because of the much longer perturbation life time of CO_2. Hence, the second ratio on the right hand side in Eq. (4) will be larger than one and asymptotically approach one as H approaches infinity.

The overall ratio in Eq. (4) will thus become slightly larger than one when CH_4 is gas X, since the ratio for CH_4 (the first ratio on the right hand side) will reach unity faster than the ratio for CO_2 since methane as a much shorter life time.[1] This explains the result shown in Fig. 1.

If, on the other hand, the atmospheric life time of gas X is relatively long, such as for SF_6, the first ratio would approach unity more slowly than the ratio for CO_2 (for the time horizons studied here). Thus the IGTP-to-GWP ratio becomes less than one for SF_6.[2] Rotmans and den Elzen (1992) also noted that the ratio IGTP-to-GWP is higher

[1] For the very interested reader, it might be worthwhile to note that since CO_2 (approximately) decays with a series of exponential time constants, one of which is much shorter than the decay time of methane, this affects the IGTP-GWP ratio so that it becomes less than unity the first few years.

[2] The ratio becomes higher than unity for time horizons so long that most of SF_6 has decayed away (see also Peters et al., 2011). The reason for that is that parts of the CO_2 will remain for longer time horizons than SF_6. The atmospheric perturbation life time of CO_2 cannot be

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for gases with short life times, although they used slightly different concepts for IGTP and GWP.

For N_2O the IGTP-to-GWP ratio is initially less than one, but then becomes larger than one. This is explained by the fact that for short time horizons, an emissions pulse of N_2O decays from the atmosphere more slowly than CO_2, but on longer time horizons an emissions pulse of N_2O decays more rapidly than CO_2.

Explanation of the results in the sensitivity analysis

If there is no inertia in the climate system, it can be seen right away from Eq. (4) that the IGTP-to-GWP ratio will become equal to one (both ratios in the right hand side of Eq. (4) are equal to unity, since there is no difference between transient and equilibrium temperature response). Now, when considering the inertia of the climate system (that results from the heat capacity of the oceans) the IGTP-to-GWP ratio will deviate from unity. The larger the inertia is (as a result of higher climate sensitivity or higher diffusivity), the more the deviation of the ratio from unity will be. This is the fundamental reason behind the results of the sensitivity analysis presented in Figs. 2 and 3.

5 Conclusions

This paper addresses similarities between different metrics to compare greenhouse gases, in particular between GWP and IGTP. A near equivalence between IGTP and GWP is demonstrated. IGTP and GWP are near equivalent in two ways: (1) they are identical if there is no thermal inertia in the climate system; (2) they are asymptotically equal when the time horizon approaches infinity. The values differ only by in the

captured in a single time constant. Most of the CO_2 decays with time constants shorter than the life time of SF_6. For this reason, the ratio is lower than one for the time horizon studied in this paper.

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cases studied here, by at most some 10 %. Our research corroborates the results by Peters et al. (2011) and Rotmans and den Elzen (1992) who used different modeling approaches.

In addition, we also carry out a sensitivity analysis with respect to the climate sensitivity and the heat diffusivity, and the found that the difference between IGTP and GWP increases with the inertia in the climate system (higher inertia stems either from a higher climate sensitivity or from higher heat diffusivity).

The near equivalence between IGTP and GWP can be understood in physical terms. Since the integration of the temperature response must be (over infinitely long time horizons) proportional to the integrated radiative forcing of a pulse emission, it follows that GWP and IGTP are similar. The analysis here has thus focused on how they deviate on shorter time scales. In Appendix A it is shown that SGTP and IGTP are identical under assumptions about linearity.

Given that these metrics (GWP, SGTP and IGTP) are either equivalent or near equivalent, the exact choice of these metrics is of less importance for abatement decisions. Hence, there is no compelling reason why SGTP or IGTP should be chosen over GWP.

While it matters little for abatement policy whether IGTP, SGTP or GWP is used when making trade-offs, it is more important to decide whether society should use a metric based on time integrated effects such IGTP and GWP, a snapshot metrics as GTP, or metrics where both economics and physical considerations are taken into account (see Manne and Richels, 2001; O'Neill, 2000; Shine, 2009; Azar and Johansson, 2011 and Johansson, 2012). Of equal importance is the question of how to choose the time horizon, regardless of the chosen metric. For these questions, value judgments are needed which cannot solely be answered by the scientific community.

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Appendix A

Demonstrating the equivalence between sustained global temperature potential (SGTP) and the integrated global temperature change potential (IGTP)

Shine et al. (2005) state that SGTP and GWP are "near equivalent". They show that this near equivalence holds numerically for certain time horizons. However, they also state that "the near equivalence of the GWP and GTP_S at 100 years does not guarantee equivalence at other time horizons". (They refer to SGTP as GTP_S). In their paper, it is not explicitly stated in what sense SGTP and GWP are "near-equivalent", and why. Here, we attempt to do that.

In the first step we show that SGTP(H) and IGTP(H) are identical metrics under certain conditions that will be defined below.

SGTP is the temperature response of a gas X emitted at a constant (sustained) rate ($1\,\mathrm{kg\,yr^{-1}}$) divided by the temperature response following sustained emissions of CO_2 ($1\,\mathrm{kg\,yr^{-1}}$). Or more formally,

$$\mathrm{SGTP}(H) = \frac{\mathrm{ASGTP}_X(H)}{\mathrm{ASGTP}_{\mathrm{CO_2}}(H)} \tag{A1}$$

where ASGTP(H) is the absolute temperature response at time H following sustained emissions during the period $0 < t < H$, and defined as the integrated effect of a the temperature response of a series of pulse emissions, i.e.

$$\mathrm{ASGTP}_X(H) = \int_0^H T_{\mathrm{P}X}(H,\tau)\,\mathrm{d}\tau. \tag{A2}$$

Here T_{PX} (H, τ) is the temperature response at time H from a pulse emission at time τ.

Now, assume that the temperature response is the same regardless of when in time the pulse emission occurs, i.e. we assume linearity in gas cycles, radiative forcing

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calculations and the temperature model, which is standard when estimating GWPs. If so,

$$T_{PX}(H, \tau) = T_{PX}(H - \tau). \quad \text{(A3)}$$

We insert expression (Eqs. A3 into A2), and then, through the variable substitution, $t = H - \tau$, ASGTP can be rewritten as

$$\text{ASGTP}_X(H) = \int_0^H T_{PX}(H - \tau)\, d\tau = \int_0^H T_{PX}(t)\, dt. \quad \text{(A4)}$$

Thus, $\text{ASGTP}_X(H)$ is equal to the integrated temperature response from a pulse emission of gas X. Since it holds for gas X, it will hold for all gases (including CO_2). Thus, it follows that

$$\text{SGTP}(H) = \frac{\text{ASGTP}_X(H)}{\text{ASGTP}_{CO_2}(H)} = \frac{\int_0^H T_{P,X}(t)\, dt}{\int_0^H T_{P,CO_2}(t)\, dt} = \text{IGTP}(H). \quad \text{(A5)}$$

Thus, SGTP is identical to IGTP when linearity in greenhouse gas cycles and temperature responses are assumed. Thus, the results reported in this paper for the relationship between IGTP and GWP also hold for SGTP (given these linearity assumptions), i.e. SGTP is "near equivalent" to GWP for reasons explained in Sect. 4 of this paper.

Hence, that SGTP and IGTP are equal measures might seem surprising given that SGTP is an end-point measure whereas the IGTP is an integrative measure but it follows from the fact that SGTP is a measure of the temperature change at one point in time from a sustained emission, i.e. it is the integrated temperature effect of a series a pulse emissions.

If the background concentration is changing SGTP and IGTP will only approximately be equal.

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Appendix B

Modeling historic temperatures

Our UDEBM was run using historic radiative forcing data from the representative concentration pathway scenarios (see Meinshausen et al., 2011) using different combinations of values for the climate sensitivity (CS), the vertical heat diffusivity (K) and the total radiative forcing from aerosols. It can be seen in Fig. B1 that the model fairly well reproduces the historic global mean surface temperature change (as estimated by NASA GISS, 2011) for each set of parameter combinations. When changing either the climate sensitivity or the vertical heat diffusivity, changes in the aerosol forcing are required to maintain a good fit with historic temperatures. There is significant uncertainty in the aerosol forcing, but our assumptions are well within the estimated range (Forster et al., 2007).

Appendix C

IGTP and GWP values

In Tables C1–C3 we summarize the GWP and IGTP values for CH_4, N_2O and SF_6 for different assumptions on the climate sensitivity and effective vertical heat diffusivity.

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Appendix D

Demonstrating the asymptotic equivalence between IGTP and GWP

Let us assume the following simple one-box energy balance model of the climate system with thermal inertia:

$$C \frac{\mathrm{d}T}{\mathrm{d}t} = F(t) - T/\lambda \tag{D1}$$

where C is the heat capacity of the layer that should be heated, $F(t)$ the radiative forcing, T the increase in temperature and λ the climate sensitivity (as above). Assume now that one kg of gas X is emitted, and that it decays exponentially with a life time of τ_X, $F_X(t) = F_X e^{-t/\tau_X}$. We then obtain the temperature response to a pulse emission of gas X as

$$T_{PX}(t) = \frac{\tau_X \lambda F_X}{\tau_X - \lambda C} \left(e^{-t/\tau_X} - e^{-t/\lambda C} \right). \tag{D2}$$

Integrating over the temperature response gives

$$\int_0^H T_{PX}(t)\,\mathrm{d}t = \frac{\tau_X \lambda F_X}{\tau_X - \lambda C} \left\{ \tau_X \left(1 - e^{-H/\tau_X} \right) - \lambda C \left(1 - e^{-H/\lambda C} \right) \right\}. \tag{D3}$$

We notice that if H goes to infinity, the integral converges towards $\tau_X \lambda F_X$. This can be understood in the following way: since the integral over the temperature response is the same as the absolute SGTP for a gas X, we may instead of integrating the temperature response, directly consider the equilibrium temperature response of a sustained pulse of gas X. This temperature response is equal to the equilibrium increase in the atmospheric mass of gas X multiplied by the forcing per kg multiplied by the climate sensitivity, i.e. $\tau_X \lambda F_X$.

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For CO_2, the atmospheric "decay" function is somewhat more complicated. We assume that it can be approximated by a sum of exponential functions and a constant term (see IPCC, 2007), so that the radiative forcing at time t from the emission of kg of CO_2 at time 0 is equal to

$$F_{CO_2}(t) = F_{CO_2}\left(\alpha_0 + \sum_{i=1}^{3} \alpha_i \, e^{-t/\tau_{CO_2 i}}\right). \quad \text{(D4)}$$

If so, the temperature following a pulse emission would be

$$T_{P,CO_2}(t) = \lambda \, F_{CO_2}\left\{\alpha_0\left(1 - e^{-\frac{t}{\lambda C}}\right) + \sum_{i=1}^{3} \frac{\alpha_i \, \tau_i}{\tau_i - \lambda \, C}\left(e^{-\frac{t}{\tau_i}} - e^{-\frac{t}{\lambda C}}\right)\right\}. \quad \text{(D5)}$$

Integrating over the temperature response gives

$$\int_0^H T_{P,CO_2}(t)\,\mathrm{d}t = \lambda \, F_{CO_2}\left\{\alpha_0\left(H + C\,\lambda\left(e^{-\frac{H}{\lambda C}} - 1\right)\right)\right.$$

$$\left. + \sum_{i=1}^{3} \frac{\alpha_i \, \tau_i}{\tau_i - \lambda \, C}\left\{\tau_i\left(1 - e^{-\frac{H}{\tau_i}}\right) - \lambda \, C\left(1 - e^{-\frac{H}{\lambda C}}\right)\right\}\right\}. \quad \text{(D6)}$$

Now calculate IGTP/GWP in the limit $H \to \infty$

$$\lim_{H\to\infty} \frac{\mathrm{IGTP}(H)}{\mathrm{GWP}(H)} = \lim_{H\to\infty} \frac{\frac{\tau_X \lambda F_X}{\tau_X - \lambda C}\left\{\tau_X\left(1-e^{-\frac{H}{\tau_X}}\right) - \lambda C\left(1-e^{-\frac{H}{\lambda C}}\right)\right\}}{\lambda F_{CO_2}\left\{\alpha_0\left(H + C\lambda\left(e^{-\frac{H}{\lambda C}} - 1\right)\right) + \sum_{i=1}^{3} \frac{\alpha_i \tau_i}{\tau_i - \lambda C}\left\{\tau_i\left(1-e^{-\frac{H}{\tau_i}}\right) - \lambda C\left(1-e^{-\frac{H}{\lambda C}}\right)\right\}\right\}} = 1. \quad \text{(D7)}$$
$$\frac{\tau_X F_X\left(1-e^{-\frac{H}{\tau_X}}\right)}{F_{CO_2}\left\{\alpha_0 H + \sum_{i=1}^{3} \alpha_i \tau_i\left(1-e^{-\frac{H}{\tau_i}}\right)\right\}}$$

Acknowledgements. Financial support from the Swedish Energy Agency and Carl Bennet AB is gratefully acknowledged.

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Table 1. IGTP estimates for CH_4, N_2O and SF_6 (with GWP values in brackets).

Time horizon	CH_4	N_2O	SF_6
20	77 (72)	284 (289)	15 800 (16 200)
100	29 (25)	301 (298)	22 100 (22 800)
500	8 (8)	162 (153)	32 000 (32 600)

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Table C1. GWP and IGTP values for CH_4.

Time horizon	GWP	IGTP				
		CS = 3 K = 2	CS = 4.5 K = 2	CS = 2 K = 2	CS = 3 K = 4	CS = 3 K = 0.5
20	72	77	78	76	77	76
100	25	29	30	27	29	28
500	8	8	9	8	9	8

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Table C2. GWP and IGTP values for N_2O.

Time horizon	GWP	IGTP				
		CS = 3 K = 2	CS = 4.5 K = 2	CS = 2 K = 2	CS = 3 K = 4	CS = 3 K = 0.5
20	289	284	283	285	284	284
100	298	301	301	300	301	300
500	153	162	166	160	165	158

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Table C3. GWP and IGTP values for SF_6.

Time horizon	GWP	IGTP				
		CS = 3 K = 2	CS = 4.5 K = 2	CS = 2 K = 2	CS = 3 K = 4	CS = 3 K = 0.5
20	16 200	15 800	15 700	15 800	15 800	15 800
100	22 800	22 100	21 900	22 300	22 000	22 300
500	32 600	32 000	31 800	32 200	31 900	32 300

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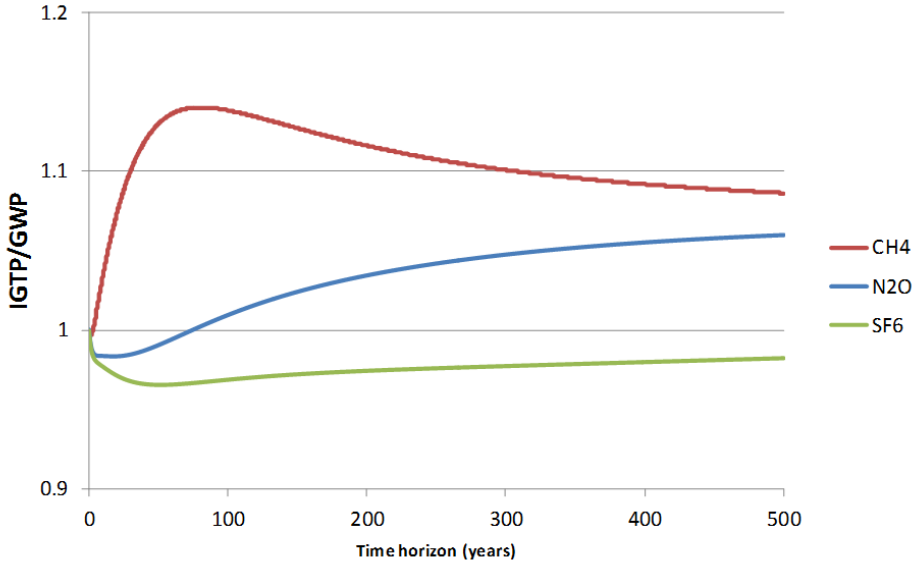

Fig. 1. IGTP/GWP ratio for CH_4, N_2O and SF_6 depending on the time horizon *H* (in years).

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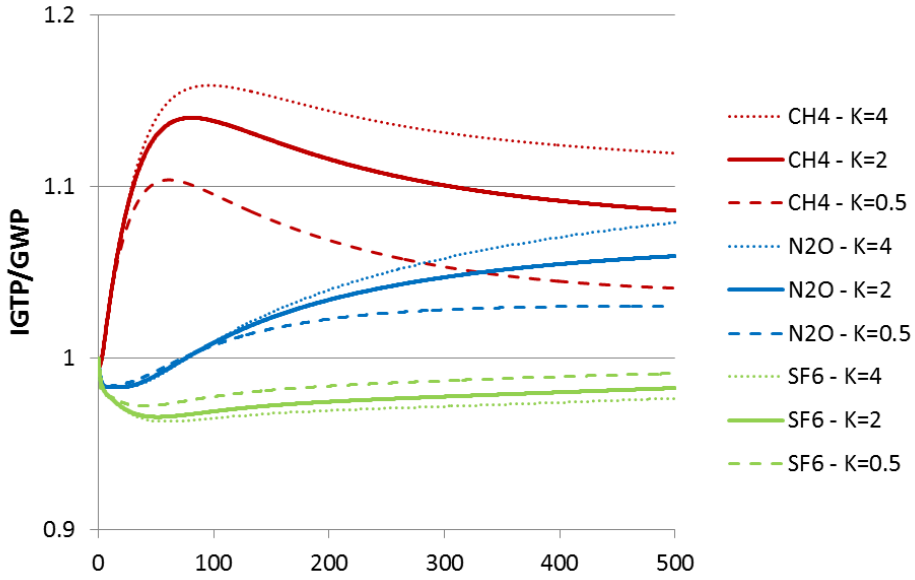

Fig. 2. IGTP/GWP ratio for CH_4 depending on effective vertical heat diffusivity. K, the vertical heat diffusivity, is given in $cm^2\ s^{-1}$.

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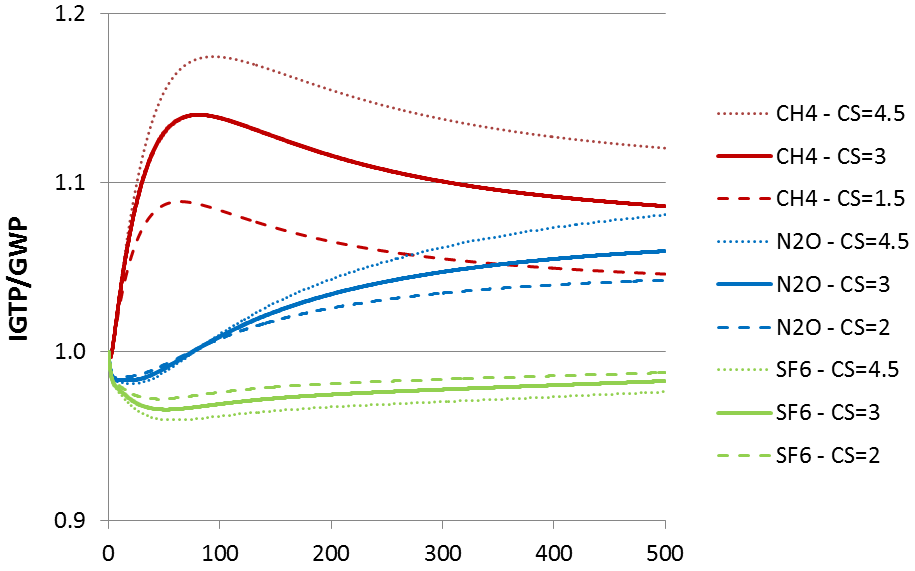

Fig. 3. IGTP/GWP ratio for CH_4, N_2O and SF_6 depending on climate sensitivity. CS, the climate sensitivity, is given in kelvins per CO_2 equivalent doubling.

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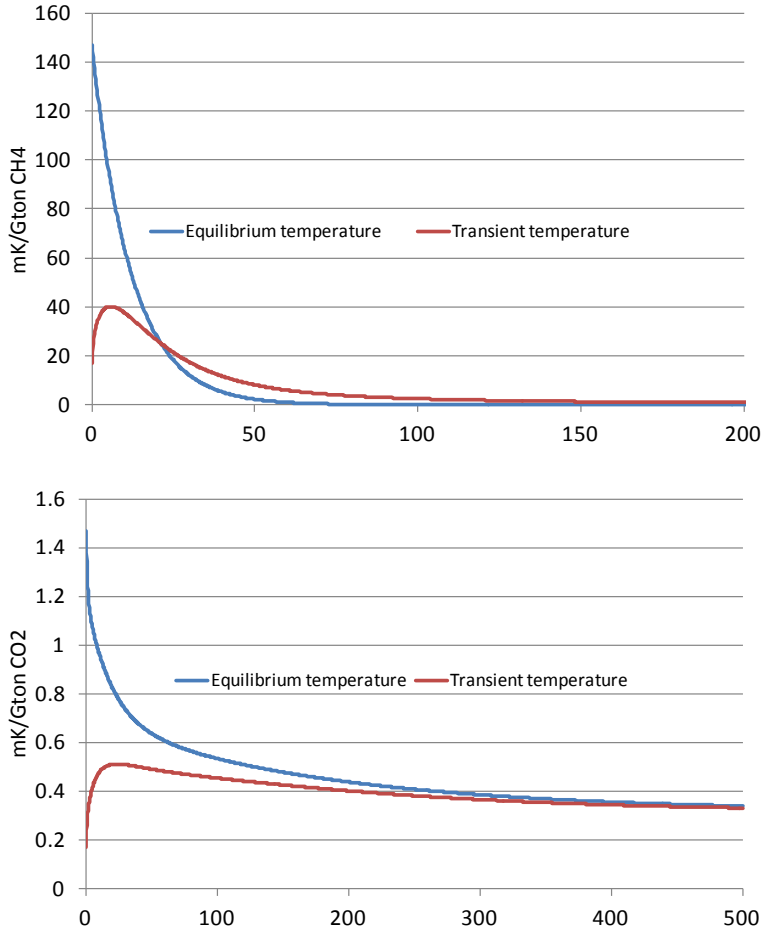

Fig. 4. Equilbrium and transient temperature response for a pulse emission of CH_4 (upper panel) and CO_2 (lower panel).

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Relationship between metrics to compare greenhouse gases

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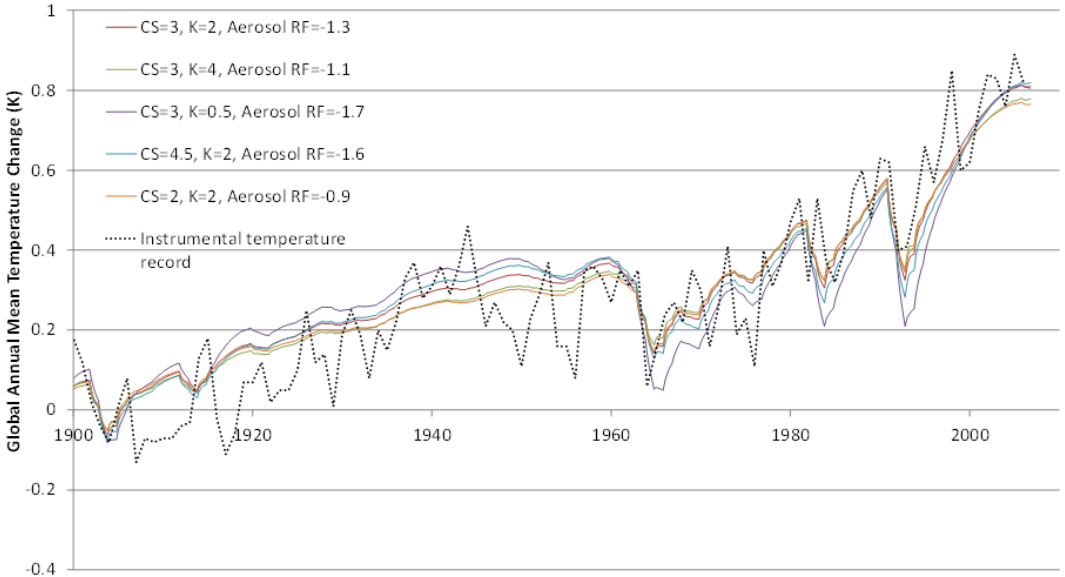

Fig. B1. Modeled and observed historic global mean surface temperature change. CS = climate sensitivity, K = the vertical heat diffusivity. The aerosol forcing is given in W m^{-2} for the year 2005. Historic estimates for the aerosol forcing are scaled linearly with this value.

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