Entropy Production of Soil Hydrological Processes and its Maximisation
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Reviewer's Comments

The authors present an account of the application of the maximum entropy production hypothesis / heuristic to soil hydrology, which includes the effect of vegetation. The analysis contains some interesting facets and is worthy of investigation. The analysis, however, contains a number of flaws in its theoretical formulation, some of them quite fundamental. The reader therefore can have no confidence in the methodology applied or its results. The errors must be corrected, and the analyses repeated and rewritten, before this work can be considered for publication.

Comments

1. The equations in the manuscript are presented in an unusual style and syntax, which could cause confusion, especially to soil engineers. They must be converted to standard nomenclature. In particular:

   - The authors should not use dots as multiplication signs, unless they wish to indicate the dot product of vectors (which then should be in bold).

   - Some quantities are labelled using multiple symbols, e.g. RH for relative humidity (p3), and *dsdT, zT* and *f*netΓ in (11) (p 4). While this style is used in non-rigorous fields such as agricultural science, it causes tremendous confusion and must never be used: each quantity should only have one symbol. If necessary, superscripts, subscripts and accents can be used for further identification. If *dsdT* is indeed a derivative, it should be written correctly, either in the notation of Newton or Leibnitz.

2. Many parameters are not defined appropriately and/or differ from standard usage in soil mechanics. The following should be corrected and/or more detailed justification should be provided for their use:

   - R_V in (1) is not the ideal gas constant 8.314 J K^{-1} mol^{-1}, since it has units of m^2 s^{-2} K^{-1} = J kg^{-1} K^{-1}. It is therefore applicable only to air. How do you account for its variability due to the variable moisture content (hence density and molecular mass) of moisture-laden air?

   - The "matric potential" Ψ_M (p3) is normally expressed as a quantity Ψ_M / g, in units of metres of liquid head, rather than in J kg^{-1} = m^2 s^{-2}; see Bear & Bachmat (1991), pp 338+; Corey (1994), p 80. Similarly, the potential μ is normally given as the piezometric head μ / g. It may not be important, but why due you employ this usage?

   - Neither the definition nor the units of Θ_{soil}, the relative water content, are stated (e.g. by volume or by mass)? Is it a volume fraction? How does it relate to the water saturation *S*, and why is your analysis not cast in terms of the latter? Do you consider the irreducible component? At very least, a conversion formula should be provided.

- The van Genuchten correlation (3) is merely one of several correlations commonly used. It does not appear to be consistent with the form reported in Bear & Bachmat (1991), pp 344+ and Corey (1994), p48:

$$S_e = \frac{S_w - S_{w0}}{1 - S_{w0}} = \left[\frac{1}{1 + (\alpha h)^n} \right]^m \qquad \text{with } h = \frac{\Psi_M}{g}$$

where h is the capillary head, S_e is the effective water saturation, S_w is the water saturation and S_{w0} the irreducible water saturation. Inversion gives:

$$\Psi_M = gh = \frac{g}{\alpha} \left[\frac{1}{S_e^{1/m}} - 1 \right]^{1/n}$$

Since Θ_{soil} is undefined, it is not clear how your version should be expressed, but have you mistakenly taken the reciprocal of Θ_{soil} in the van Genuchten correlation? Also, why do you reverse the sign of the (negative) potential Ψ_M ?

- A more advanced form of the van Genuchten correlation (3) is given in Bear & Bachmat (1991), pp 344+, which includes the irreducible air saturation. Would this be more useful?

- Finally, the van Genuchten correlation (3) is a simple monotonic curve, which does not account for wetting-drying hysteresis. Why did you choose this curve, and how would you account for hysteretic effects?

- The vegetation potential equation (4) is rather bewildering. If Θ_{veg} is the volume fraction of water in the vegetation, it can never exceed 1 (as confirmed by your Figure 3), so is there any need for the maximum in (4)? Is Ψ_{PWP} a modified matric potential, with the same units as Ψ_M, or something different (it looks like a piezometric head)? If so, why use the symbol Ψ ?

3. The individual formulations of entropy production in §2.2 appear correct (or at least reasonable), but it is not clear whether all processes have been included: it is essential to redraw Figure 1 in the form of a true engineering flow diagram, showing all individual compartments (control volumes), all possible flows between these compartments, and the control surface (boundary) of the entire control volume. Bidirectional flows should be distinguished from unidirectional flows. Thus:

- The infiltration flow path (surface water $\rightarrow$ soil) is not shown in Figure 1, but its EP is included.

- In contrast, one precipitation path (atmosphere $\rightarrow$ surface water) is shown, but its EP is omitted.

- Another precipitation path (atmosphere $\rightarrow$ river) is not shown and its EP is not included.

- The path of bare soil condensation (atmosphere → soil; e.g. dew, frost) is not shown; or is it included in the precipitation?

- The paths of surface runoff evaporation (surface water → atmosphere) and river evaporation (river → atmosphere) are not shown.

- Where bi-directional flow is possible (e.g. root water uptake; baseflow), the entropy production must be positive for both flow directions.

The authors really must account for all possible flows between all compartments. Furthermore, at steady state, the EP of a system is given *either* by the sum of EPs due to internal flows, *or* by the sum EPs through the external boundary (Ozawa *et al.*, 2001). It is not permissible to count the EPs due to both internal and external flows. Thus:

- The EP of the river path (river → out) is not shown in Figure 1, but is included in the summation. This is an external flow, so should it be included?

- Alternatively, if the authors wish to consider only external flows, they should add only the EPs due to the river path (river → outside) and the delivery of humid air to the system (outside → atmosphere).

4. The definition of the potential μ_{rain} should be more clearly explained. Also, rain is a flow path, not a compartment; a more appropriate symbol might be $\mu_{surface\ water}$ or $\mu_{overland\ water}$.

5. Several of the entropy production terms depend upon linearised (Onsager-like) force-flux relations (5), (7), etc, the transport terms of which are used as free parameters (Figure 4). Are there any other free transport parameters, e.g. of transpiration, or other free variables, such as vegetation density?

6. The discretisation scheme used should be explained more clearly. Are each of the EP terms in §2.2 calculated for each grid element, or only in total? Are local or global potentials used? This comes back to the question of local versus global EP, exhibited in the different formulations of Dewar (2003, 2005) and Niven (2009).

7. Figure 3 is confusing, since it combines two effects; it would be better to separate them.

Due to the above serious problems, I did not examine the findings in §4.

References

Bear, J. & Bachmat, Y. (1991) Introduction to Modeling of Transport Phenomena in Porous Media, Kluwer Academic Publishers, Dordrecht, Netherlands.

Corey, A.T. (1994), "Mechanics of Immiscible Fluids in Porous Media", 3rd ed., Water Resources Publications, Highlands Ranch, CO, 252 pp.

Dewar, R.C. (2003) Information theory explanation of the fluctuation theorem, maximum entropy production and self-organized criticality in non-equilibrium stationary states, J. Phys. A: Math. Gen. 36: 631-641.

Dewar, R.C. (2005) Maximum entropy production and the fluctuation theorem, J. Phys. A: Math. Gen. 38: L371-L381.

Niven, R.K. (2009) Steady state of a dissipative flow-controlled system and the maximum entropy production principle, Physical Review E 80(2): 021113.

Ozawa H, Shimokawa S, Sakuma H. 2001. Thermodynamics of fluid turbulence: A unified approach to the maximum transport properties. Physical Review E 64, 026303