Comment on "The problem of the second wind turbine – a note on a common but flawed wind power estimation method," by F. Gans, L.M. Miller, and A. Kleidon (Earth Syst. Dynam. Discuss., 1, 103-114, doi:10.5194/esdd-1-103-2010, 2010)

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Gans et al. (2010) (hereinafter GMK10) claim that previous methods of estimating global or regional wind power, whereby energy is extracted in a wake volume downwind of a turbine, are flawed because such methods don't conserve energy. They then claim that their method of determining wind energy dissipation under the assumption of channel-flow is more accurate. These claims are both incorrect, as the relevant papers criticized by GMK10 (Archer and Jacobson, 2005; Archer and Caldeira, 2009; Lu et al., 2009; Sta. Maria and Jacobson, 2009), implicitly account for the physics of wind flow past turbines, thus energy sources and sinks, more realistically than do GMK10. In fact, GMK10's method is unphysical and unrepresentative of the real atmosphere, thus not relevant for atmospheric analysis, particularly for the evaluation of other studies. GMK10 also make a significant error in a calculation they derive from one of the papers they criticize, which they rely on to draw their conclusion. With this error, they inaccurately imply that turbines used to power the world will significantly affect global energy available. Even ignoring this error, their paper does not show that powering the world with wind will reduce the globally-averaged power available at hub height by more than a small amount (<1%) or the kinetic energy integrated over the boundary-layer by more than a trivial amount (<0.1%). In sum, the analysis of GMK10 does not change the conclusions of the studies that they criticize nor provide a better method of calculating wind energy or energy dissipation in the atmosphere.

The main error with GMK10 is the setup of their experiment. They make the unphysical assumption that wind turbines are in a closed channel with no sources or sinks of energy in the channel other than turbine dissipation. As a result, they ignore wind production and loss within a wake downwind of the turbine due to basic terms in the momentum equation. They also ignore the thermodynamic energy equation, which accounts for energy advection, adiabatic compression/expansion, and diabatic heating of the air. They further ignore the conversion of kinetic energy to electricity and then to heat; instead, they assume that the kinetic energy converted by wind turbines just disappears. Therefore, GMK10 do not represent real flows or real sources and sinks of energy correctly, so their method cannot conserve energy.

First, GMK10 erroneously assume that the only physical process affecting the wind is energy loss by the turbine and transfer of the remaining energy not dissipated by one turbine to the hub of another. They ignore all other terms in the momentum equation responsible for production, loss, and transport of winds, including the pressure gradient force (PGF), the apparent Coriolis force (ACF), local acceleration (based on velocity gradients), friction, and turbulent flux divergence. It can be shown (e.g., Figure 1) that when these effects are accounted for, winds lost at a turbine are regenerated downwind by the PGF and ACF in the horizontal and by the vertical transport of horizontal momentum. The ultimate source of energy for these terms is sunlight, whose differential heating creates pressure gradients that drive winds. GMK10 ignore this renewable energy as a source of regenerated winds within a wake downwind of turbine.

With respect to Figure 1, as wind speeds first decrease within a wake, the vertical gradient in wind speed increases, increasing the downward turbulent flux of faster winds from aloft to hub height and decreasing the downward flux from hub height to the surface, replenishing winds downwind of each turbine. Similarly, the horizontal pressure gradient force continuously acts on the winds at a given height and contributes establishing the quasi-geostrophic balance among the PGF, ACF, friction, and turbulent flux divergence terms.

Figure 1. Average west-east wind speed across a control cross-sectional area of sized 3 x D in the south-north direction (where D is turbine diameter) and 1000 m (boundary-layer height) vertically, upwind and downwind of four sequential turbines after 2.3 days of a time-dependent simulation in a 1-D west-east model initialized with winds at rest (0 m/s). Winds are generated over time by a combination of a south-north pressure gradient (assumed in this example to be 5 hPa / 100 km), the apparent Coriolis force, local west-east acceleration of the predicted wind speed gradient by the west-east wind, and vertical transfer of horizontal momentum. Four 5 MW, 126-m diameter (D) rotor turbines with hub heights of 100 m are staggered for illustration purposes at lengths 19D. The vertical molecular plus turbulent diffusion coefficient in the boundary layer is assumed to be 150 m^2/s (a case of strong convection). The wind speed at 2 x hub height, for purposes of calculating vertical transfer of horizontal momentum is estimated by interpolating a power law profile between the geostrophic wind speed, the wind speed at hub height, and the surface wind speed (0 m/s). Each turbine is assumed to reduce the wind speed over the swept area of the turbine (πD^2/4) by 50%. Because the control cross-sectional area for the turbine is 3D multiplied by the height of the boundary layer (1000 m), the turbine swept area is only ~3.3% of the control area, and the average wind speed reduction over this control cross-sectional area immediately downwind of the turbine is only about ~1.65% (rather than 50%). The grid spacing for this plot was 1 m; the time step used was 0.01 s.
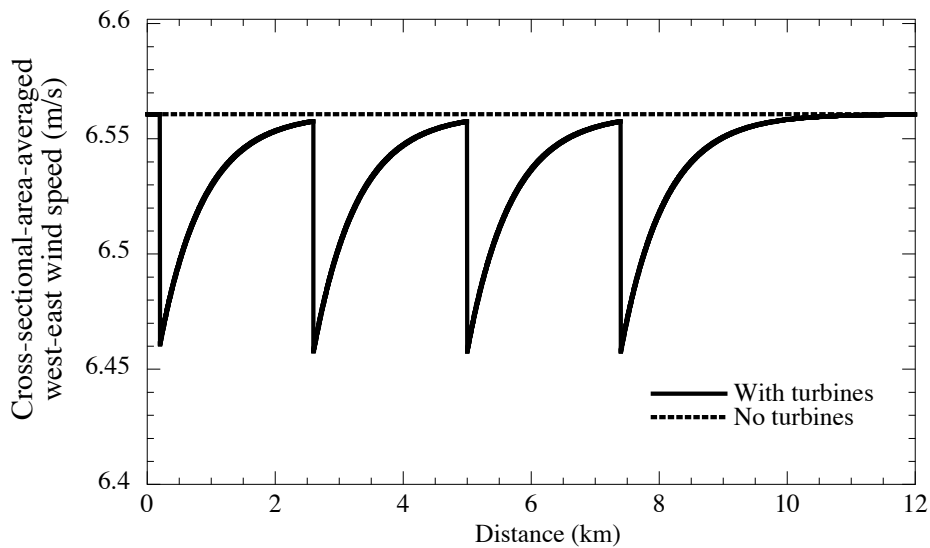
The simple model in Figure 1 is by no means complete nor indicative of actual spacing required to optimize wind farm energy output, as the most rigorous and ultimately accurate method of simulating the effect of wind turbines on the atmosphere is with a three-dimensional model at high resolution accounting for many more terms than used for Figure 1. However, it illustrates plainly that GMK10's assumption that wind is not regenerated at all within a wake downwind of a turbine is not realistic. The distance downwind that regeneration occurs varies with meteorological conditions, but the fact is, the wind always regenerates at some point so long as horizontal pressure gradients and vertical wind speed gradients exist. Thus, the loss of energy in the atmosphere due to wind turbines occurs within some wake volume. Within that volume, wind speeds first decrease then increase, eventually converging to the background wind speed. This was the assumption in the papers criticized by GMK10, and this assumption is physical. GMK10, on the other hand, claim that wind speeds at the end of the wake can only be lower than the wind speeds upstream of a turbine; thus, they believe it is not possible for winds to regenerate in the wake. This assumption is invalid.

In sum, the major error in GMK10 is their assumption of channel flow for wind energy, where no sources or sinks of energy occur within the channel. In their scenario, they assume that the only exchange of energy is loss by turbine dissipation. This would be the case for turbines in a river of water, but such is not the case for atmospheric flows.

GMK10 argue further that several papers make the same unrealistic channel-flow assumption as they themselves do. However, this is not the case. For example, Sta. Maria and Jacobson (2009), hereinafter SJ09, assume that the atmosphere is three-dimensional (as it is in reality). The losses of energy due to a wind turbine occur in a wake volume. Within the wake volume, wind is regenerated so that by the end of the

wake, the wind speed has regenerated to that upstream of the turbine (e.g., Figure 1). One can argue that the wake volume is too large or too small (since the wake volume was simply estimated based on historic spacing in actual wind farms), but one cannot claim that the methodology accounts for energy less accurately than does the methodology of GMK10 since GMK10 ignore all sources of kinetic energy aside from the initial wind entirely. In SJ09, energy is lost in the wake volume, and that is accounted for, and energy gained with increasing distance from the turbine within the wake volume is due to regeneration of winds from solar energy producing pressure gradients, which ultimately produce all winds.

GMK10 make a further error in their claim that the calculation in SJ09 that the world's energy needs can be met by extracting only 0.007% of the kinetic energy of the lowest 1 km of the atmosphere implies that the wind power in the lowest km must be 242,000 TW. This number, which is a factor of over 140 higher than available world wind energy, was surmised by Gans, Miller, and Kleidon but is not derivable from a correct interpretation of the data in SJ09. The authors did not attempt to replicate the calculations in SJ09 (Eq. 17 in particular), for if they did, they could not have made such a significant error. Instead of contacting the authors of SJ09 to clarify their uncertainty, they submitted it for publication and broadcast it at a scientific meeting, calling it a "flawed methodology" that "violates atmospheric energetics."

GMK10's error arose because they confused power at a given height (hub height) with energy integrated over the boundary layer. For example, they divided their estimate of world power extracted by wind turbines at hub height (e.g., 100 m) needed to supply their estimate of power demand of 17 TW, by the fractional loss in boundary-layer-integrated energy, 0.00007, calculated in SJ09. This gives the erroneous 242,000 TW number. However, wind power extracted by a turbine through its swept area centered at hub height is not a boundary-layer averaged value; nor does power (energy per unit time) even have the same units as energy. Power extracted by a turbine is the energy per unit time passing through a turbine swept area that is converted to electric power. Atmospheric kinetic energy lost in the wake of turbine is the kinetic energy averaged over the wake volume as if the turbine were absent minus that over the same volume as if the turbine were present. The kinetic energy in the presence of the turbine must account not only for the loss of energy due to its extraction by the turbine, but also production along the wake, as demonstrated in Figure 1 here. The boundary-layer-averaged fractional loss in kinetic energy is that difference multiplied by the ratio of the volume of all turbine wakes to the volume of the boundary layer, all divided by the kinetic energy of the entire boundary layer. In sum, power extraction alone cannot be equated properly with energy extraction minus production and hub-height power extraction cannot be equated with boundary-layer-integrated energy change. GMK10 mixed up two unrelated parameters, resulting in a factor of >140 error in their result.

Below, we clarify how the numbers in SJ09 were derived and their correct interpretation.

First SJ09 assume the use of 1.5 MW turbines with D=77-m diameter rotors and array spacing of 3D x 10D (SJ09, Eq. 14). The mean observed wind speeds over oceans and land worldwide from data referenced in SJ09 were given as 8.6 m/s and 4.8 m/s, respectively. Applying these mean wind speeds to the capacity factor equation CF=0.087 V(m/s) – P(kW)/D(m)^2 (SJ09, Eq. 20) for 1.5 MW turbines gives CFs of 0.495 and 0.165, respectively.

Second, the surface area of the Earth is 5.106×10^{14} m^2. Thus, the total Earth could theoretically fit 2.87×10^9 1.5 MW turbines with spacing of 3D x 10D, with 29% over land and 71% over the ocean.

Third, before turbine presence, the total power in the wind at 100 m deliverable for energy assuming 1.5 MW turbines over water and land can be estimated theoretically as 2.87×10^9 turbines x 1500 kW x (0.71 x 0.495 + 0.29 x 0.165) ~1700 TW, where 0.71 and 0.29 are the fraction of world area as ocean and land, respectively, and 0.495 and 0.165 are the CFs over ocean and land, respectively (derived above). This is almost exactly the annual world wind power at 100-m hub height (~1700 TW) calculated independently with a 3-D model run at 1.5 degree horizontal resolution in which instantaneous predicted wind speeds in each grid cell were applied to the power curve of a 5 MW Re-Power turbine (126-m diameter rotor), with cut-in wind speed of 3.5 m/s and cut-out wind speed of 30 m/s (Jacobson and DeLucchi, 2010). Such dual-independent calculations of world wind power (one based on a model and one based on world wind data observations) are more accurate than that from the zero-dimensional back-of-the-envelope calculation from the 1950s relied on by GMK10. The 1700 TW is not even close to the 242,000 TW GMK10 claim are determined by SJ09.

Fourth, world delivered power in 2030 for all purposes is calculated to be 16.9 TW (Jacobson and DeLucchi, 2009, 2010). However, a conversion from fossil fuels to electricity and electricity-produced hydrogen reduces 2030 power demand for all purposes by 30% to 11.5 TW (Jacobson and Delucchi, 2009, 2010). As such, the maximum power at 100 m necessary for extraction is only 11.5 TW/1700 TW=0.0068 (0.68%). Accounting for transmission and conversion losses of ~15% increases this loss to 0.78% (and increases the corresponding energy required). In reality, however, plans to power the world call for only up to 50% of all world power to be supplied by wind (e.g., Jacobson and DeLucchi, 2009, 2010), so only 0.39% of the world's wind power at 100 m is needed in practice. In either case, converting the world to 50% or 100% wind consumes < 1% of the world's wind power at hub height.

Fifth, SJ09 state that 10 million 1.5 MW turbines (D=77 m diameter rotor) would be needed to replace all fossil-fuel energy with wind if the turbines were exposed to average wind speeds of 7 m/s. However, this number of turbines was estimated under the assumption that all energy for all purposes would be converted to electricity with the same efficiency as electric vehicles. An updated and more likely

scenario is conversion to a combination of electricity and electricity-produced hydrogen, which would require 3.8 million 5 MW turbines at a mean wind speed of 7.75 m/s to power 50% of the world's energy in 2030 (Jacobson and DeLucchi, 2009). For 100% conversion to wind in 2030, this translates roughly to 22 million 1.5 MW turbines at 7 m/s. Below, we will use the 10 million turbine number from SJ09 first, but then scale the final result to 22 million turbines.

Sixth, the surface area for spacing required to power the world for all purposes with 10 million 1.5 MW turbines operating in 7 m/s winds is only 3D x 10D x 10^7 turbines / Area of the Earth = 0.00348, or 0.348% of the surface area of the Earth. The relative area for 22 million 1.5 MW turbines is 0.766%.

Seventh, the ratio of the wake volume of one wind turbine to the total control volume of the turbine over a 3D*10D surface area, extended to h=1000 m (boundary-layer height), is $13\pi D/36h$ (Eq. 15 of SJ09), or 0.0874 for D=77 m. Thus, the volume of the boundary layer affected by wind turbine energy dissipation for 10 million turbines relative to the volume of the Earth's boundary layer is ~ 0.00348 * 0.0874 = 0.000304. That for 22 million turbines is 0.000669.

Finally, the change in kinetic energy of the wind in the bottom 1 km of the atmosphere over all land globally due to powering the world on 10 million 1.5 MW turbines in the presence of 7 m/s wind speed relative to the kinetic energy in the wind in the bottom 1 km of the atmosphere worldwide is approximately 0.000304 x $(7^2 - 6.09^2) / (0.71 * 8.6^2 + 0.29*4.8^2)$ ~ 0.000061, or 0.0061% (where mass in the numerator and denominator are assumed to cancel). In this equation, 8.6 m/s is the mean wind speed over the world's oceans; 4.8 m/s is the mean wind speed over land, 7 m/s is the mean wind speed upstream of wind turbines, 6.09 m/s is the mean wind speed in turbine wakes, and 0.71 and 0.29 are the fractional areas of the world covered by ocean and land, respectively. The 6.09 m/s estimate is based on the average wind speed reduction of ~13% in the wake of a turbine, derived from velocity deficit versus wind speed, weighted by a Rayleigh distribution of the wind speed about a mean of 7 m/s, in SJ09. The 0.0061% loss in energy worldwide is lower than the average of 0.008% calculated in SJ09, Fig. 11, at 7 m/s upstream speed simply because the estimate here is based on a calculation at the mean wind speed rather than on a calculation over a Rayleigh distribution, as in Eq. 17 of SJ09. Thus, the numbers in SJ09 (0.008% reduction in world energy in the boundary layer at 7 m/s or 0.007% at 8 m/s) are more accurate, but the number here is easier to demonstrate by example. With 22 million turbines, the numbers increase to 0.018% and 0.015%, respectively. Even if the wake volume affected by wind turbine energy dissipation were low by a factor of 5, the energy lost in the bottom 1 km of the global atmosphere due to powering 100% of the world's energy for all purposes on wind at 7 m/s upstream speed would be <0.1%, still a trivial loss considering the benefits of such a conversion; namely, eliminating global warming and more than 2.5 million air pollution deaths per year. Considering further that wind need power only 50% of the world, these numbers can be divided by two.

In sum, the methodology in GMK10 is unphysical, not accounting for real processes in the atmosphere or for the conversion of wind-derived electricity to heat. As such, it does not conserve energy. The papers criticized by GMK10 implicitly account better for energy, although no method is perfect. Further, GMK10's conclusions rely on a significant error in their interpretation of another paper. Finally, GMK10 do not show that powering the world with wind will reduce the globally-averaged wind power available at hub height by any more than a small amount (<1%) or the kinetic energy integrated over the boundary-layer by more than a trivial amount (<0.1%). As such, the GMK10 results neither alter the conclusions of papers they criticize nor provide relevant information about the effects of wind turbines on the atmosphere.

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