Earth Syst. Dynam. Discuss., 1, 103–114, 2010
www.earth-syst-dynam-discuss.net/1/103/2010/
doi:10.5194/esdd-1-103-2010

Earth System
Dynamics
Discussions

The problem of the second wind turbine – a note on a common but flawed wind power estimation method

F. Gans[1], **L. M. Miller**[1,2], **and A. Kleidon**[1]

[1]Biospheric Theory and Modeling Group, Max Planck Institute for Biogeochemistry, Hans-Knoell-Strasse 10, 07745, Jena, Germany
[2]International Max-Planck Research School for Earth System Modeling, Bundesstr. 53, 20146, Hamburg, Germany

Received: 29 April 2010 – Accepted: 27 May 2010 – Published: 8 June 2010

Correspondence to: F. Gans (fgans@bgc-jena.mpg.de)

Published by Copernicus Publications on behalf of the European Geosciences Union.

Abstract

Several recent wind power estimates suggest how this renewable resource can meet all of the current and future global energy demand with little impact on the atmosphere. These estimates are calculated using observed wind speeds in combination with specifications of wind turbine size and density to quantify the extractable wind power. Here we show that this common methodology is flawed because it does not account for energy removal by the turbines that is necessary to ensure the conservation of energy. We will first illustrate the common but flawed methodology using parameters from a recent global quantification of wind power in a simple experimental setup. For a small number of turbines at small scales, the conservation of energy hardly results in a difference when compared to the common method. However, when applied at large to global scales, the ability of radiative gradients to generate a finite amount of kinetic energy needs to be taken into account. Using the same experimental setup, we use the simplest method to ensure the conservation of energy to show a non-negligble decrease in wind velocity after the first turbine that will successively result in lower extraction of the downwind turbines. We then show how the conservation of energy inevitably results in substantially lower estimates of wind power at the global scale. Because conservation of energy is fundamental, we conclude that ultimately environmental constraints set the upper limit for wind power availability at the larger scale rather than detailed engineering specifications of the wind turbine design and placement.

1 Introduction

Several recent studies have quantified large-scale or global wind power availability by extrapolating kinetic energy availability from measured wind speeds (Archer and Jacobson, 2005; Archer and Caldeira, 2009; Magdalena and Jacobson, 2009; Lu et al., 2009; Liu et al., 2008; Leithead, 2007). These studies follow a common methodology as follows:

1. Observations of wind velocities v are derived for a large spatial area either from a collection of surface stations (e.g. Archer and Jacobson, 2005), satellite measurements (e.g. Liu et al., 2008), or reanalysis data (e.g. Archer and Caldeira, 2009).

2. Technical specifications for a turbine are then used to specify the rotor height, rotor-swept area, and velocity dependent power characteristics.

3. Generated power resulting from the extracted kinetic energy is calculated for a single turbine by $P_{ex} = \frac{1}{2}\rho C_f A_{\mathrm{rotor}} v^3$, where C_f is the capacity factor dependent on the selected turbine, ρ is the air density, and A_{rotor} is the rotor-swept area.

4. The area is then populated with wind turbines at a density limited by engineering constraints related to turbine wake turbulence with the basic assumption that wind turbines do not influence the wind velocity field outside their specific influential volume (e.g. Magdalena and Jacobson, 2009).

Other studies by Keith et al. (2004) and used global climate models to account for the effect of wind power extraction by large-scale wind farms, but they used this effect primarily to demonstrate the influence of large-scale wind power extraction on the atmospheric flow and the global climate and not for quantifying the effect of extraction on wind power availability.

Here, we will first show that the common method violates the fundamental law of energy conservation when applied to a simplified setting. This is done by presenting an experimental setup that uses only velocities and turbine influence areas to estimate wind power. We use the turbine specifications presented in Magdalena and Jacobson (2009), not to focus on the flaws of this particular study, but rather to highlight the severe limitations of this methodology now common in recent scientific publications (Archer and Jacobson, 2005; Archer and Caldeira, 2009; Magdalena and Jacobson, 2009; Liu et al., 2008; Lu et al., 2009; Leithead, 2007). We then use the conservation of energy as a starting point to derive extracted wind power to the same experimental design as a best-case scenario. This best-case scenario shows that the conservation of energy leads to substantially lower estimates of wind power availability with greater number of wind turbines because of the influence of increasing numbers of wind turbines on the wind velocity. We close with the application of these considerations to wind power availability at the global scale.

2 Methods and results

To compare the two methodologies, we define a fixed experimental tunnel setup with a length ($L = 100$ km), width ($w = 200$ m), and height ($h = 785$ m), approximately the thickness of the atmospheric boundary layer (Fig. 1). This results in a tunnel cross-sectional area of $A_{\text{tunnel}} = wh = 160{,}000\ \text{m}^2$. In our idealized case, there is no frictional loss between the air and the walls of the tunnel. We accelerate the air at the entrance of the tunnel to a fixed velocity of $v_0 = 10\ \text{m/s}$. The kinetic energy input per unit time P_{in} is the kinetic energy density of the moving air multiplied by the volume flux J_V defined as:

$$P_{\text{in}} = \frac{1}{2}\rho v_0^2 \cdot J_V = \frac{1}{2}\rho A_{\text{tunnel}} v_0^3 \qquad (1)$$

Assuming a constant inlet air density of $\rho = 1.275\ \text{kg/m}^3$, $P_{\text{in}} = 100$ MW of mechanical power is required to accelerate the tunnel of air to 10 m/s.

2.1 Fixed-velocity method

To quantify the wind power extraction potential using fixed, measured velocities and turbine influence areas, we use the 2 MW Tjaereborg turbine specifications described in Magdalena and Jacobson (2009). In their study, the wind turbine (diameter, $D = 60$ m) creates a cone-like volume of air (length, $l = 10 \cdot D = 600$ m, and radii of $1 \cdot D = 60$ m and $3 \cdot D = 180$ m). This cone-shaped influence volume is situated in the tunnel as shown in Fig. 1 so the turbine does not interfere with the tunnel walls or downwind turbines. Given the available tunnel length of 100 km and influence length, $l = 10 \cdot D = 600$ m, 150 individual turbines can be placed every 660 m without one turbine affecting another according to (Magdalena and Jacobson, 2009).

In this methodology, each turbine is driven by the same wind velocity of 10 m/s because the downwind turbine is located outside the influence volume. Given the capacity factor of $C_f = 0.56$, the power extraction of one turbine $P_{\text{ex,i}}$ is calculated as

$$P_{\text{ex,i}} = \frac{1}{2} C_f \rho A_{\text{rotor}} v_0^3 = C_e \cdot P_{\text{in}} \tag{2}$$

with an effective capacity $C_e = A_{\text{rotor}}/A_{\text{tunnel}} \cdot C_f$. For each turbine, Eq. (2) yields an extracted power of $P_{\text{ex,i}} \approx 1$ MW.

The total extracted power by N turbines of this method is:

$$P_{\text{ex}} = N \cdot C_e \cdot P_{\text{in}} \tag{3}$$

Figure 2 shows how the extracted power has a linear dependence on the number of installed turbines. For $N = 150$ we get a total extracted power of $P_{\text{ex}} \approx 150$ MW.

According to Magdalena and Jacobson (2009), the kinetic energy reduction within the influenced volume is $\Delta\text{KE}/\text{KE} \approx 0.4$. This leads to a wake velocity of $v_{\text{wake}} = 7.7$ m/s within the cone-shaped influence volume with the surrounding air maintaining the velocity of 10 m/s. The kinetic energy loss in the tunnel is now calculated as:

$$\text{KE}_{\text{loss}} = \frac{\Delta\text{KE}}{\text{KE}} \cdot \frac{S_{\text{wake}}}{S_{\text{tunnel}}} = 2.8\% \tag{4}$$

where S_{wake} is the accumulated turbine influencial volume and S_{tunnel} is the tunnel volume. This kinetic energy loss is then presumably replenished from an infinite reservoir of wind power outside the system boundary, because otherwise this setup would violate the starting assumption that the velocity of the flow v_0 is fixed.

In summary, this experimental setup based on the methodology and assumptions of Magdalena and Jacobson (2009) is able to extract 150 MW of wind power from an initial influx of 100 MW, while also simultaneously decreasing the kinetic energy within the tunnel by 2.8% and allowing 100 MW to exit the tunnel. Although these power quantities and wind velocities are specific to our experimental design, this methodology clearly violates the conservation of energy in general.

2.2 Energy conservation method

To estimate the best case scenario for how much power can be extracted from the flow, we start with the conservation of energy. We still neglect friction and other inefficiencies. Conservation of energy at the first turbine requires that the inflow of kinetic energy P_{in} at the first turbine balances the extracted power $P_{\text{ex},1}$ and the outflow of kinetic energy P_{out} behind the influence volume: $P_{\text{in}} = P_{\text{ex},1} + P_{\text{out},1}$. Knowing that $P_{\text{ex},1} = C_e \cdot P_{\text{in}}$ as above (see Eq. 2), this leads to $P_{\text{out},1} = (1 - C_e) \cdot P_{\text{in}}$ in wind power behind the first turbine. This reduction in power requires that the velocity of the mean flow behind the first turbine is reduced, so at the second turbine, the incoming power is reduced to $P_{\text{in},2} = P_{\text{out},1} = (1 - C_e) \cdot P_{\text{in}}$. Here we assume that C_e is constant and does not decrease due to reduced velocities resulting in an optimistic estimate. Extending this to more turbines means that each turbine reduces the amount of power available for all downwind turbines to extract, each reducing the power by $P_{\text{in},i}/P_{\text{in},i-1} = 1 - C_e$. Or, expressed differently, turbine i (where i is the downwind turbine count) extracts power $P_{\text{ex},i}$ from the mean flow as:

$$P_{\text{ex},i} = C_e \cdot (1 - C_e)^{i-1} \cdot P_{\text{in}} \tag{5}$$

Note that the extracted power per turbine as given by Eq. (5) corresponds to the common method (cf. Eq. 2) only for the first turbine ($i = 1$), but is less for any of the following engines ($i > 1$). In the setup chosen here, the wind power of the flow behind the first turbine is reduced by 1%, resulting in a tiny decrease in velocity from 10 to 9.97 m/s. Even in a laboratory environment this tiny decrease would be difficult to detect (especially when using a “tunnel” specification representative of the global atmosphere), but it is a natural consequence of the conservation of energy.

The total extracted power by N turbines of this method is:

$$P_{\mathrm{ex}} = P_{\mathrm{in}} \left(1-(1-C_e)^N\right) \tag{6}$$

The total extracted power by enforcing energy conservation corresponds closely to the common method for small N, but with increased turbine count, the total extracted power increases at a slower rate and approximates the total power entering the tunnel (see Fig. 2). This, of course, is the logical consequence of the conservation of energy that we ought to expect.

Using the same experimental layout as before, $N = 150$ turbines extract 78 MW of power and 22 MW of power is exiting the tunnel (Fig. 1). With this method, energy conservation is maintained (78 MW+22 MW= 100 MW).

In summary, the conservation of energy dictates that for a given inflow of kinetic energy, the kinetic energy of the mean flow behind the first turbine in the tunnel must be reduced due to the extraction of power by the turbine, even if it amounts to only a tiny fraction. Hence, the second turbine downwind (and any other turbine) cannot extract as much power as the first turbine. We would therefore expect substantially less wind power to be available for extraction than estimated by the common method, with the limit being ultimately set by the inflow of kinetic energy and *not* by the number of turbines, their characteristics, or their placement. This limit is a specification of the environment, which in our case is the specified inflow of $P_{\mathrm{in}} = 100$ MW. Hence, this upper limit is therefore independent on turbine or placement specifications.

2.3 Implications for large-scale wind power estimates

To demonstrate that the conservation of energy is of relevance for large-scale estimates of wind power, we briefly consider the methodology with respect to the previously published estimates. Using well-established energetics of the atmosphere, the global generation of wind power is estimated to be around 900 TW (Lorenz, 1955; Peixoto and Oort, 1992). Less known thermodynamic derivations show that this rate of power generation is at a maximum value given the present-day radiative forcing gradients, as demonstrated by simple theoretical considerations (Lorenz, 1960), box models (Paltridge, 1978; Lorenz et al., 2001) and general circulation model simulations (Kleidon et al., 2003, 2006). Of course, not all of the 900 TW could ever be captured by wind turbines. First, extraction of wind power is always going to compete with the natural processes of turbulent dissipation. Second, it would seem impossible to derive technological means that could extract all of the generated kinetic energy of the global atmosphere. In any case, this estimate of 900 TW sets the uppermost limit on available wind power (Gustavson, 1979). With global energy demand in 2009 being 17 TW (EIA, 2009), human energy consumption corresponds to about 2% of total wind power of the global atmosphere.

The most recent global wind power quantification by Magdalena and Jacobson (2009) claims that the world's energy needs can be met by extracting only 0.007% of the kinetic energy of the lowest 1 km of the atmosphere. This statement translates into an assumption of wind power in the lowest 1km of the atmosphere of ≈242 000 TW, which is roughly twice the global amount of absorbed solar radiation (122 000 TW). Clearly, the study by Magdalena and Jacobson (2009) not only violates atmospheric energetics, it even violates the planetary energy balance!

A similarly flawed methodology was used by Archer and Caldeira (2009) in which they estimated the wind power of the high-altitude winds to be 1700 TW. This estimate of jet stream wind power is twice the well-established number of 900 TW of global wind power within the atmosphere, again violating the conservation of energy in the process of generating, extracting and dissipating kinetic energy in the atmosphere.

Both examples show that energy conservation and thermodynamic limits on generation of kinetic energy play a central role for large-scale to global-scale estimates of available wind power.

3 Conclusions

We showed in an idealized setting that the common method to estimate wind power availability to wind farms is fundamentally flawed by not considering the conservation of energy. The consequence of this lack of physical soundness implies that the common method inevitably overestimates wind power availability. The two examples cited above show that at the planetary scale, the bias is substantial. This shows how the application of seemingly precise engineering methods of turbine design and placement to large-scale estimates results in physically impossible estimates of wind power, emphasizing the dominant role on environmental limitations for large-scale estimates of wind power availability.

Such physically-flawed estimates will give worthless – and possibly even dangerous – policy advice to the urgent need to plan a sustainable human energy future. Future studies of global wind power estimates must, at a minimum, ensure that basic physical principles are being obeyed to yield sound scientific estimates.

Acknowledgement. The authors thank Ryan Pavlick for his comments which improved this manuscript. This work has been partially supported by the Helmholtz research alliance “Planetary Evolution and Life”.

The service charges for this open access publication
have been covered by the Max Planck Society.

References

Archer, C. L. and Jacobson, M. Z.: Evaluation of global wind power, J. Geophys. Res., 110, D12110, doi:10.1029/2004JD005462, 2005. 104, 105

Archer, C. L. and Caldeira, K.: Global assessment of high-altitude wind power, IEEE Trans. Energy Convers., 2, 307–319, 2009. 104, 105, 110

Energy Information Administration: International Energy Outlook 2009, United States Department of Energy – Energy Information Administration, 2009. 110

Gustavson, M. R.: Limits to wind power utilization, Science, 204, 13–17, 1979. 110

Intergovernmental Panel on Climate Change: IPCC scoping meeting on renewable energy sources – proceedings, Intergovernmental Panel on Climate Change, 2008.

Keith, D., DeCarolis, J., Denkenberger, D., Lenschow, D., Malyshev, S. L., Pacala, S., and Rasch, P.: The influence of large-scale wind power on global climate, Proc Natl Acad Sci, 101, 16115–16120, 2004. 105

Kleidon, A., Fraedrich, K., Kirk, E., and Lunkeit, F.: Maximum entropy production and the strength of the boundary layer exchange in an atmospheric general circulation model, Geophys. Res. Lett., 33, L06706, doi:10.1029/2005GL025373, 2006. 110

Kleidon, A., Fraedrich, K., Kunz, T., and Lunkeit, F.: The atmospheric circulation and states of maximum entropy production, Geophys. Res. Lett., 30(23), 223, doi:10.1029/2003GL018363, 2003. 110

Leithead, W.: Wind energy, Philos. Trans. R. Soc. Ser. A., 365, 957–970, 2007. 104, 105

Lorenz, E.: Available potential energy and the maintenance of the general circulation, Tellus, 7, 271–281, 1955. 109

Lorenz, E.: Generation of available potential energy and the intensity of the global circulation, Dynam. Climate, Pergamon Press. Oxford, UK, 86–92, 1960. 110

Lorenz, R., Lunine, J., Withers, P., and McKay, C.: Titan, Mars, and Earth: entropy production by latitudinal heat transport, Geophys. Res. Lett., 28, 415–418, 2001. 110

Liu, W. T., Tang, W., and Xie, X.: Wind power distribution over the ocean, Geophys. Res. Lett., 35, L13808, doi:10.1029/2008GL034172, 2008. 104, 105

Lu, X., McElroy, M. B., and Kiviluoma, J.: Global potential for wind-generated electricity, Proc. Natl. Acad. Sci., 106, 10933, doi:10.1073/pnas.0904101106, 2009. 104, 105

Magdalena, R. V. S. M. and Jacobson, M.: Investigating the effect of large wind farms on energy in the atmosphere, Energies, 2, 816–838, 2009. 104, 105, 106, 107, 110

Paltridge, G. W.: The steady-state format of global climate, Q. J. Roy. Met. Soc., 104, 927–945, 1978. 110

Peixoto, J. P. and Oort, A. H.: Physics of climate, American Institute of Physics, 1, 379–385, 1992. 109

Wang, C. and Prinn, R. G.: Potential climatic impacts and reliability of very large-scale wind farms, Atmos. Chem. Phys., 10, 2053–2061, doi:10.5194/acp-10-2053-2010, 2010. 105

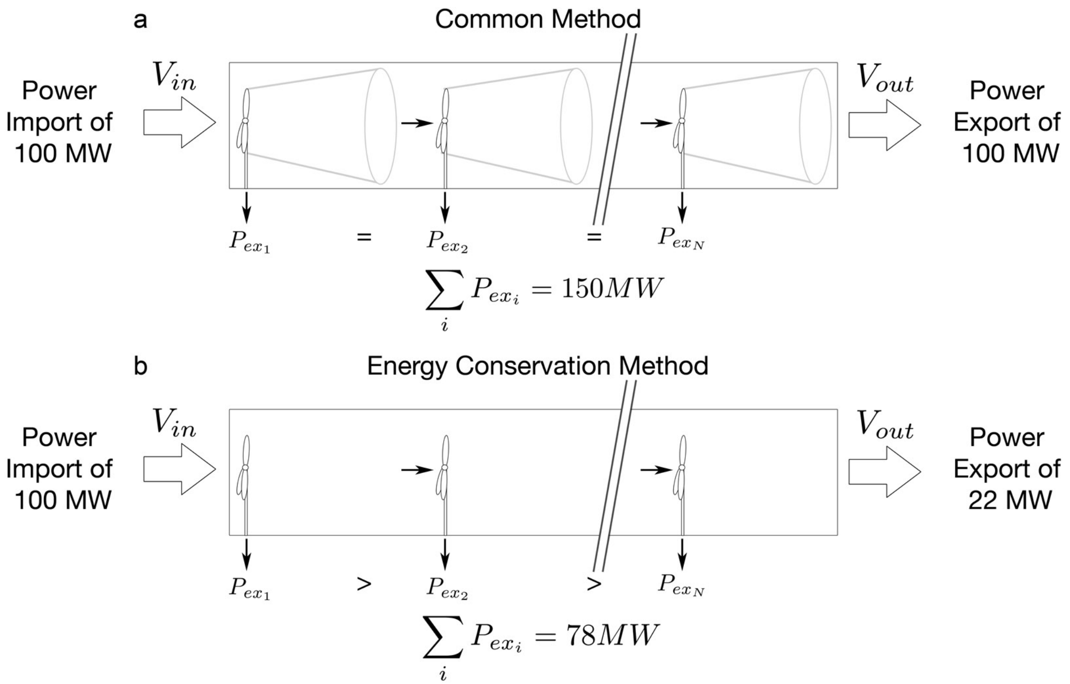

Fig. 1. This schematic illustrates **(a)** the common method to estimate wind power and **(b)** a method that conserves energy. The location and spacing of the turbines in the tunnel is the same in both methods. In case (a) the turbine does not affect the wind field outside the influential volume as indicated by the cones. In case (b) the available power decreases with turbine count ultimately limiting the extractability to the power import of 100 MW. This upper limit is a specification of the environmental setting and not of the turbine design.

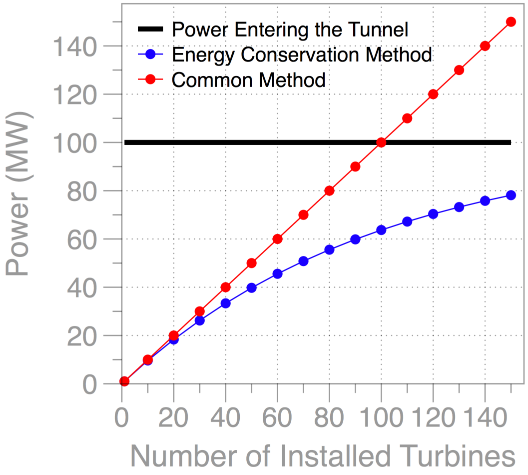

Fig. 2. Power extraction from the experimental tunnel is shown as a function of installed wind turbines from the Common Method in red, and the Energy Conservation Method in blue. The horizontal black line at 100 MW is the imported power at the inlet of the tunnel. Using the Common Method, the extracted power increases linearly with additional wind turbine installations, while the Energy Conservation Method limits extractable power to the power import into the tunnel.