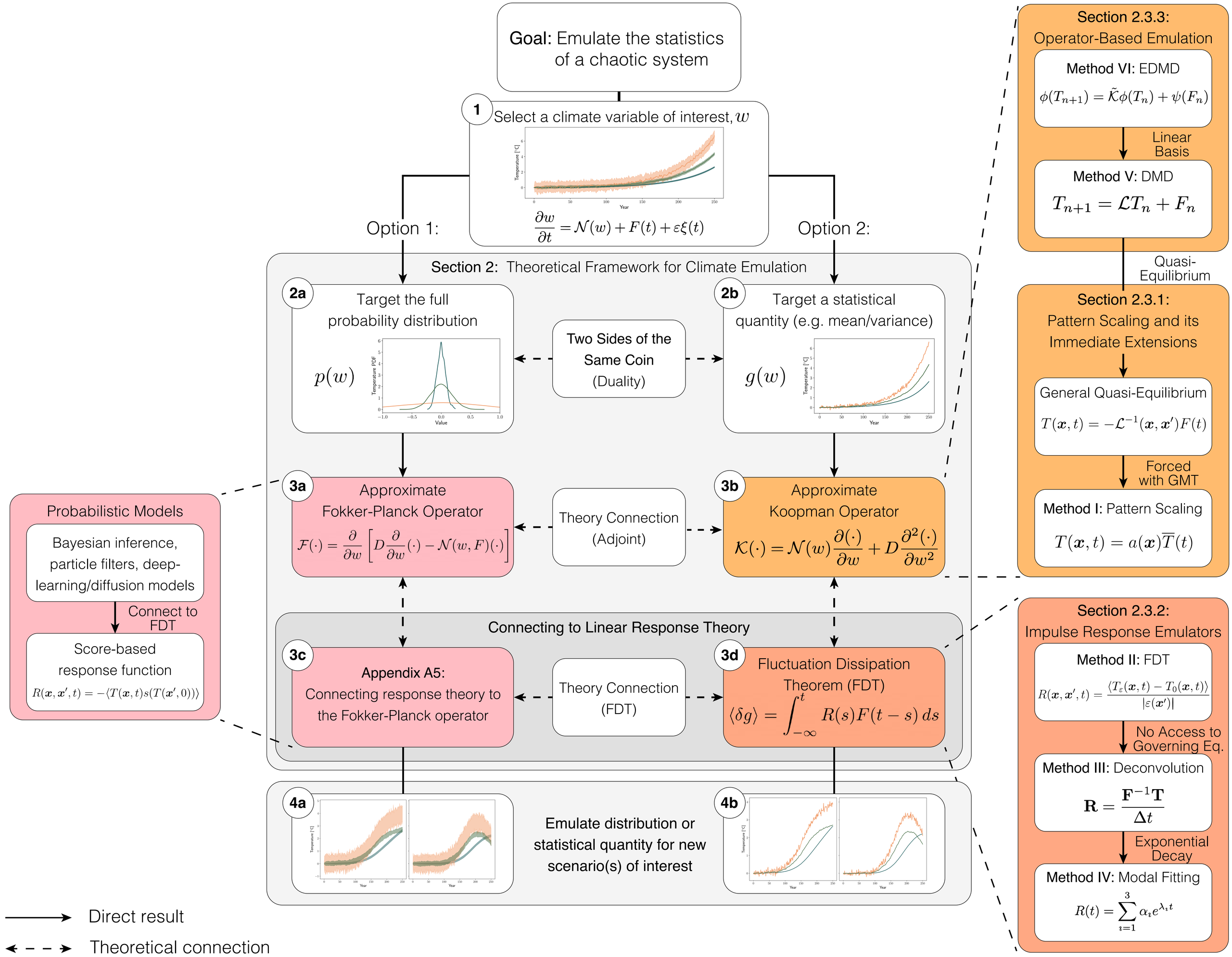
Goal: Emulate the statistics of a chaotic system
1
Select a climate variable of interest, w
$\frac{\partial w}{\partial t} = \mathcal{N}(w) + F(t) + \varepsilon\xi(t)$
Option 1:
Option 2:
Section 2: Theoretical Framework for Climate Emulation
2a
Target the full probability distribution
$p(w)$
2b
Target a statistical quantity (e.g. mean/variance)
$g(w)$
Two Sides of the Same Coin (Duality)
3a
Approximate Fokker-Planck Operator
$\mathcal{F}(\cdot) = \frac{\partial}{\partial w}\left[D\frac{\partial}{\partial w}(\cdot) - \mathcal{N}(w,F)(\cdot)\right]$
Theory Connection (Adjoint)
3b
Approximate Koopman Operator
$\mathcal{K}(\cdot) = \mathcal{N}(w)\frac{\partial(\cdot)}{\partial w} + D\frac{\partial^2(\cdot)}{\partial w^2}$
Connecting to Linear Response Theory
3c
Appendix A5: Connecting response theory to the Fokker-Planck operator
Theory Connection (FDT)
3d
Fluctuation Dissipation Theorem (FDT)
$\langle \delta g \rangle = \int_{-\infty}^{t} R(s)F(t-s)\,ds$
4a
4b
Emulate distribution or statistical quantity for new scenario(s) of interest
Probabilistic Models
Bayesian inference, particle filters, deep-learning/diffusion models
Connect to FDT
Score-based response function
$R(\boldsymbol{x},\boldsymbol{x}',t) = -\langle T(\boldsymbol{x},t)s(T(\boldsymbol{x}',0))\rangle$
Section 2.3.3: Operator-Based Emulation
Method VI: EDMD
$\phi(T_{n+1}) = \tilde{\mathcal{K}}\phi(T_n) + \psi(F_n)$
Linear Basis
Method V: DMD
$T_{n+1} = \mathcal{L}T_n + F_n$
Quasi-Equilibrium
Section 2.3.1: Pattern Scaling and its Immediate Extensions
General Quasi-Equilibrium
$T(\boldsymbol{x},t) = -\mathcal{L}^{-1}(\boldsymbol{x},\boldsymbol{x}')F(t)$
Forced with GMT
Method I: Pattern Scaling
$T(\boldsymbol{x},t) = a(\boldsymbol{x})\overline{T}(t)$
Section 2.3.2: Impulse Response Emulators
Method II: FDT
$R(\boldsymbol{x},\boldsymbol{x}',t) = \frac{\langle T_\varepsilon(\boldsymbol{x},t) - T_0(\boldsymbol{x},t)\rangle}{|\varepsilon(\boldsymbol{x}')|}$
No Access to Governing Eq.
Method III: Deconvolution
$\mathbf{R} = \frac{\mathbf{F}^{-1}\mathbf{T}}{\Delta t}$
Exponential Decay
Method IV: Modal Fitting
$R(t) = \sum_{i=1}^{3}\alpha_i e^{\lambda_i t}$
Direct result
Theoretical connection