



Supplement of

Nonlinear causal dependencies as a signature of the complexity of the climate dynamics

Stéphane Vannitsem et al.

Correspondence to: Stéphane Vannitsem (stephane.vannitsem@meteo.be)

The copyright of individual parts of the supplement might differ from the article licence.

S1 Additional figures

This supplement contains the additional figures of the manuscript. It displays the rate of information transfer of a set of climate indices on one another. Two types of analyses are made: first based on assuming linear influences between them, and second assuming that there are also nonlinear quadratic influences. The figures for the Arctic Oscillation, AO (Figs. S1-S3), the Pacific North American pattern, PNA (Figs. S3-S6), the Atlantic Multidecadal Oscillation, AMO (Figs. S7-S9), the Pacific Decadal Oscillation, PDO (Figs. S10-S12), the Tropical North Atlantic index, TNA (Figs. S13-S15) and the Quasi-Biennal Oscillation, QBO (Figs. S16-S18), are displayed here. The figures for the El Niño3.4 index and the North Atlantic Oscillation, NAO, are shown in the main document and discussed in details. The analysis of the figures in the current document is also provided in

10 the main document.



Figure S1. The rate of information transfer (left y-axis, red open circles) and the correlation (right y-axis, blue full triangles) are plotted as a function of the observables for the targeted observable (labelled TARGET in the plot): the AO. Panels (a) and (b) are for the original and Low-Frequency (LFV) time series, respectively.



Figure S2. The rate of information transfer (left y-axis, red open circles) and the correlation (right y-axis, blue full triangles) are plotted as a function of the observables for the targeted observable (labelled TARGET in the plot): the AO. Panels (a), (b) and (c) are for the original, the LFV and the high-frequency time series, respectively. The observable set is composed of 7 linear terms and 36 nonlinear quadratic terms, all listed along the x-axis. The points in black refer to the significant dependencies at the 1% level.



Figure S3. A visual representation of the linear and nonlinear influences from the set of indices to the AO for the low-frequency variability dataset. the curved plain arrows refer to the linear influence, the triple straight arrows with the same colors refer to the influence of the quadratic nonlinearities: two straight arrows are emanating from two indices, joining somewhere in between the indices, and from there the third straight arrow indicates the target. The double dashed straight (black) arrow indicates the influence of the product between the source and the target. The empty curved arrow indicates the quadratic influence of the source, while the striped curved arrow the quadratic influence of the target.



Figure S4. The rate of information transfer (left y-axis, red open circles) and the correlation (right y-axis, blue full triangles) are plotted as a function of the observables for the targeted observable (labelled TARGET in the plot): the PNA. Panels (a) and (b) are for the original and LFV time series, respectively.



Figure S5. The rate of information transfer (left y-axis, red open circles) and the correlation (right y-axis, blue full triangles) are plotted as a function of the observables for the targeted observable (labelled TARGET in the plot): the NAO. Panels (a), (b) and (c) are for the original, the LFV and the high-frequency time series, respectively. The observable set is composed of 7 linear terms and 36 nonlinear quadratic terms, all listed along the x-axis. The points in black refer to the significant dependencies at the 1% level.



Figure S6. A visual representation of the linear and nonlinear influences from the set of indices to the PNA for the low-frequency variability dataset. the curved plain arrows refer to the linear influence, the triple straight arrows with the same colors refer to the influence of the quadratic nonlinearities: two straight arrows are emanating from two indices, joining somewhere in between the indices, and from there the third straight arrow indicates the target. The double dashed straight (black) arrow indicates the influence of the product between the source and the target. The empty curved arrow indicates the quadratic influence of the source, while the striped curved arrow the quadratic influence of the target.



Figure S7. The rate of information transfer (left y-axis, red open circles) and the correlation (right y-axis, blue full triangles) are plotted as a function of the observables for the targeted observable (labelled TARGET in the plot): the AMO. Panels (a) and (b) are for the original and LFV time series, respectively.



Figure S8. The rate of information transfer (left y-axis, red open circles) and the correlation (right y-axis, blue full triangles) are plotted as a function of the observables for the targeted observable (labelled TARGET in the plot): the AMO. Panels (a), (b) and (c) are for the original, the LFV and the high-frequency time series, respectively. The observable set is composed of 7 linear terms and 36 nonlinear quadratic terms, all listed along the x-axis. The points in black refer to the significant dependencies at the 1% level.



Figure S9. A visual representation of the linear and nonlinear influences from the set of indices to the AMO for the low-frequency variability dataset. the curved plain arrows refer to the linear influence, the triple straight arrows with the same colors refer to the influence of the quadratic nonlinearities: two straight arrows are emanating from two indices, joining somewhere in between the indices, and from there the third straight arrow indicates the target. The double dashed straight (black) arrow indicates the influence of the product between the source and the target. The empty curved arrow indicates the quadratic influence of the source, while the striped curved arrow the quadratic influence of the target.



Figure S10. The rate of information transfer (left y-axis, red open circles) and the correlation (right y-axis, blue full triangles) are plotted as a function of the observables for the targeted observable (labelled TARGET in the plot): the PDO. Panels (a) and (b) are for the original and LFV time series, respectively.



Figure S11. The rate of information transfer (left y-axis, red open circles) and the correlation (right y-axis, blue full triangles) are plotted as a function of the observables for the targeted observable (labelled TARGET in the plot): the PDO. Panels (a), (b) and (c) are for the original, the LFV and the high-frequency time series, respectively. The observable set is composed of 7 linear terms and 36 nonlinear quadratic terms, all listed along the x-axis. The points in black refer to the significant dependencies at the 1% level.



Figure S12. A visual representation of the linear and nonlinear influences from the set of indices to the PDO for the low-frequency variability dataset. the curved plain arrows refer to the linear influence, the triple straight arrows with the same colors refer to the influence of the quadratic nonlinearities: two straight arrows are emanating from two indices, joining somewhere in between the indices, and from there the third straight arrow indicates the target. The double dashed straight (black) arrow indicates the influence of the product between the source and the target. The empty curved arrow indicates the quadratic influence of the source, while the striped curved arrow the quadratic influence of the target.



Figure S13. The rate of information transfer (left y-axis, red open circles) and the correlation (right y-axis, blue full triangles) are plotted as a function of the observables for the targeted observable (labelled TARGET in the plot): the TNA. Panels (a) and (b) are for the original and LFV time series, respectively.



Figure S14. The rate of information transfer (left y-axis, red open circles) and the correlation (right y-axis, blue full triangles) are plotted as a function of the observables for the targeted observable (labelled TARGET in the plot): the TNA. Panels (a), (b) and (c) are for the original, the LFV and the high-frequency time series, respectively. The observable set is composed of 7 linear terms and 36 nonlinear quadratic terms, all listed along the x-axis. The points in black refer to the significant dependencies at the 1% level.



Figure S15. A visual representation of the linear and nonlinear influences from the set of indices to the TNA for the low-frequency variability dataset. the curved plain arrows refer to the linear influence, the triple straight arrows with the same colors refer to the influence of the quadratic nonlinearities: two straight arrows are emanating from two indices, joining somewhere in between the indices, and from there the third straight arrow indicates the target. The double dashed straight (black) arrow indicates the influence of the product between the source and the target. The empty curved arrow indicates the quadratic influence of the source, while the striped curved arrow the quadratic influence of the target.



Figure S16. The rate of information transfer (left y-axis, red open circles) and the correlation (right y-axis, blue full triangles) are plotted as a function of the observables for the targeted observable (labelled TARGET in the plot): the QBO. Panels (a) and (b) are for the original and LFV time series, respectively.



Figure S17. The rate of information transfer (left y-axis, red open circles) and the correlation (right y-axis, blue full triangles) are plotted as a function of the observables for the targeted observable (labelled TARGET in the plot): the QBO. Panels (a), (b) and (c) are for the original, the LFV and the high-frequency time series, respectively. The observable set is composed of 7 linear terms and 36 nonlinear quadratic terms, all listed along the x-axis. The points in black refer to the significant dependencies at the 1% level.



Figure S18. A visual representation of the linear and nonlinear influences from the set of indices to the QBO for the low-frequency variability dataset. the curved plain arrows refer to the linear influence, the triple straight arrows with the same colors refer to the influence of the quadratic nonlinearities: two straight arrows are emanating from two indices, joining somewhere in between the indices, and from there the third straight arrow indicates the target. The double dashed straight (black) arrow indicates the influence of the product between the source and the target. The empty curved arrow indicates the quadratic influence of the source, while the striped curved arrow the quadratic influence of the target.