



*Supplement of*

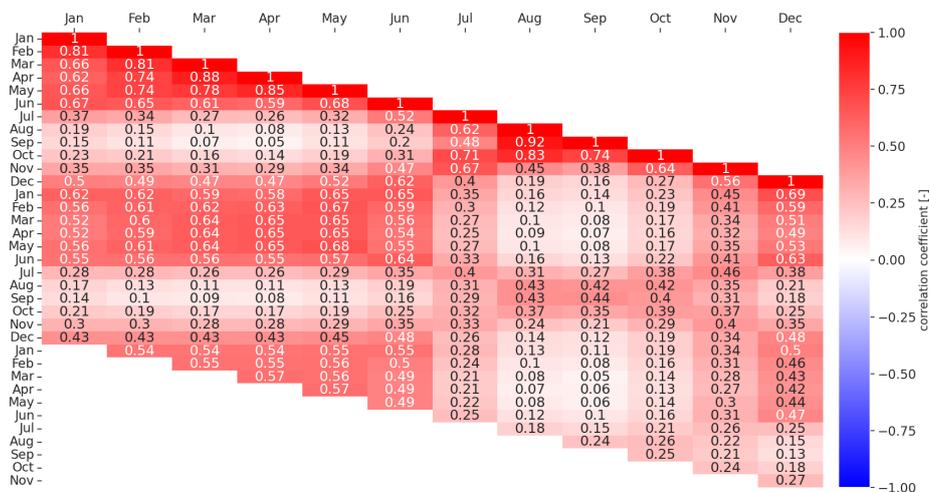
## **Ensemble design for seasonal climate predictions: studying extreme Arctic sea ice lows with a rare event algorithm**

**Jerome Sauer et al.**

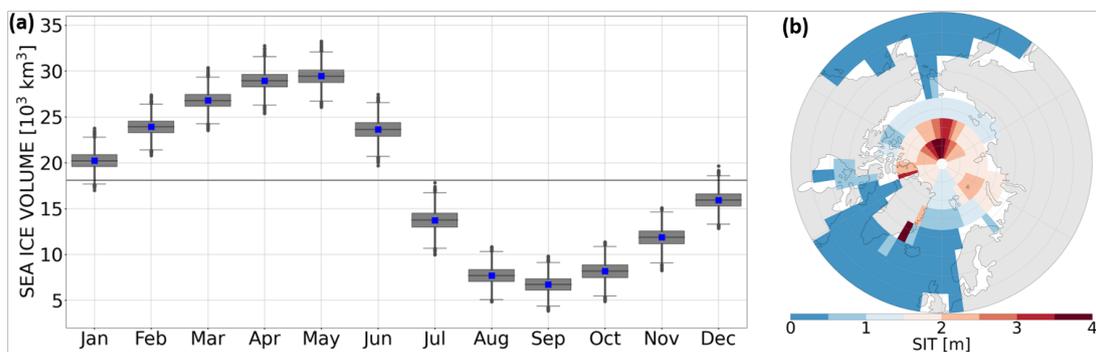
*Correspondence to:* Jerome Sauer ([jerome.sauer@uclouvain.be](mailto:jerome.sauer@uclouvain.be))

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## S1: Climatological properties of sea ice in the Planet Simulator

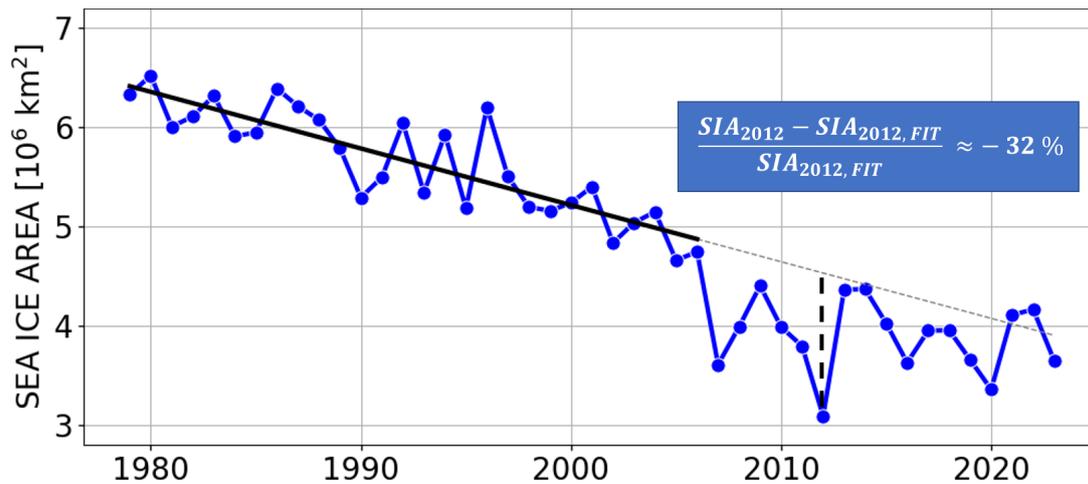


**Figure S1.** Model years 501-3500 of PlaSim-LSG control run (Sauer et al., 2024): (a) Monthly mean pan-Arctic sea ice area anomalies in the PlaSim-T21-LSG control run with fixed pre-industrial greenhouse gas conditions. Lagged correlations applied to monthly mean pan-Arctic sea ice area anomalies in PlaSim-T21-LSG with increasing lag from top to bottom up to a lag of 23 months.



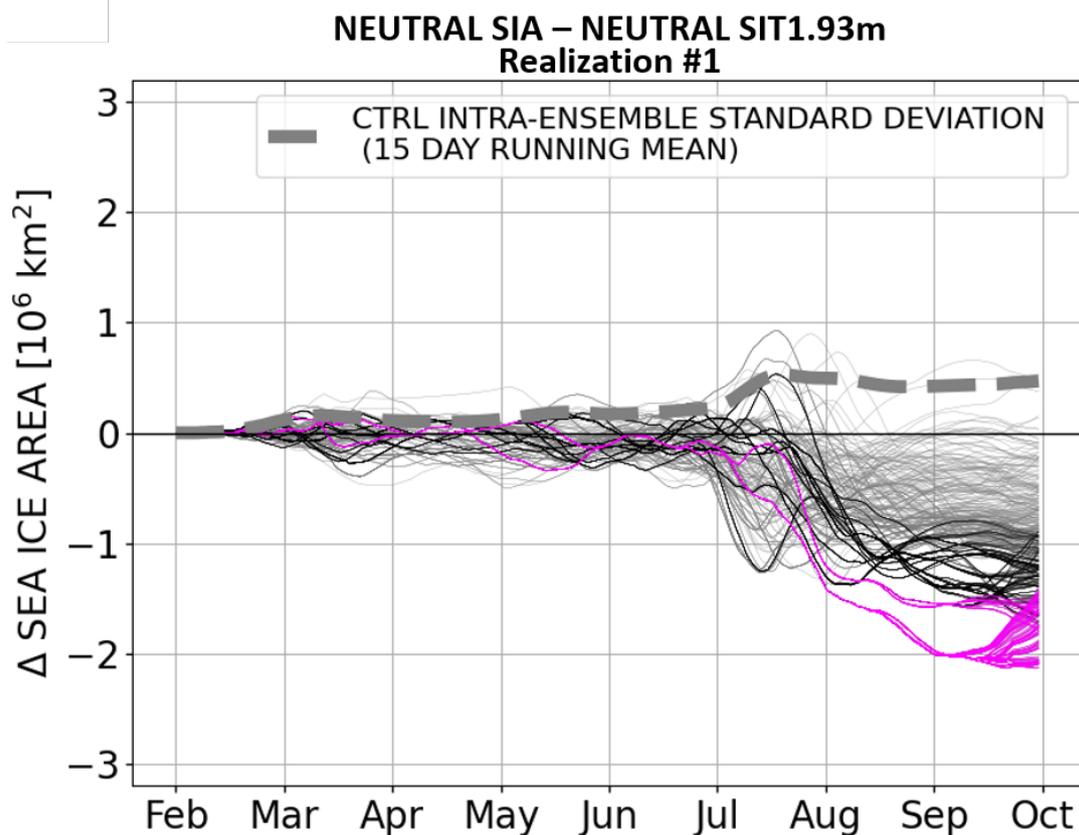
**Figure S2.** Model years 501-3500 of PlaSim-LSG control run (Sauer et al., 2024): (a) Distributions of monthly mean pan-Arctic sea ice volume [ $10^3 \text{ km}^3$ ] with respect to the 3000 control run model years. The averages and medians are given by the blue squares and the horizontal lines in the boxes. The boxes denote interquartile ranges and the maximum whisker length is defined as 1.5 times the interquartile range. The horizontal gray line shows the annual mean pan-Arctic sea ice volume. (b) Annual mean sea ice thickness [m].

## S2: Observational data

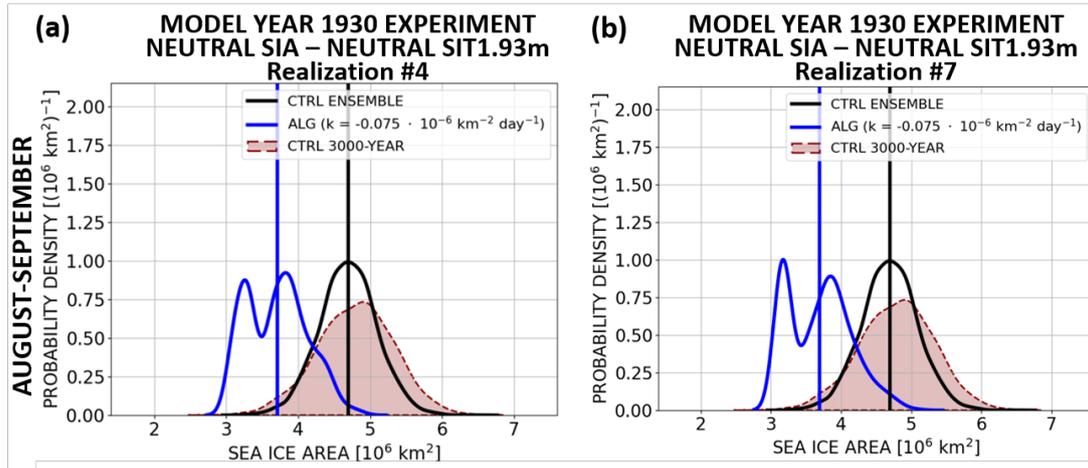


**Figure S3.** August-September mean pan-Arctic sea ice area [ $10^6 \text{ km}^2$ ] from European Organisation for the Exploitation of Meteorological Satellites (EUMETSAT) Ocean and Sea Ice Satellite Application Facility (OSI SAF) data (EUMETSAT Ocean and Sea Ice Satellite Application Facility, 2024). The black solid and dashed gray lines show a linear trend fitted (FIT) to the data of the period 1979 and 2006. Compared to the entire time series, the period 1979-2006 shows the most linear decline of the sea ice area.

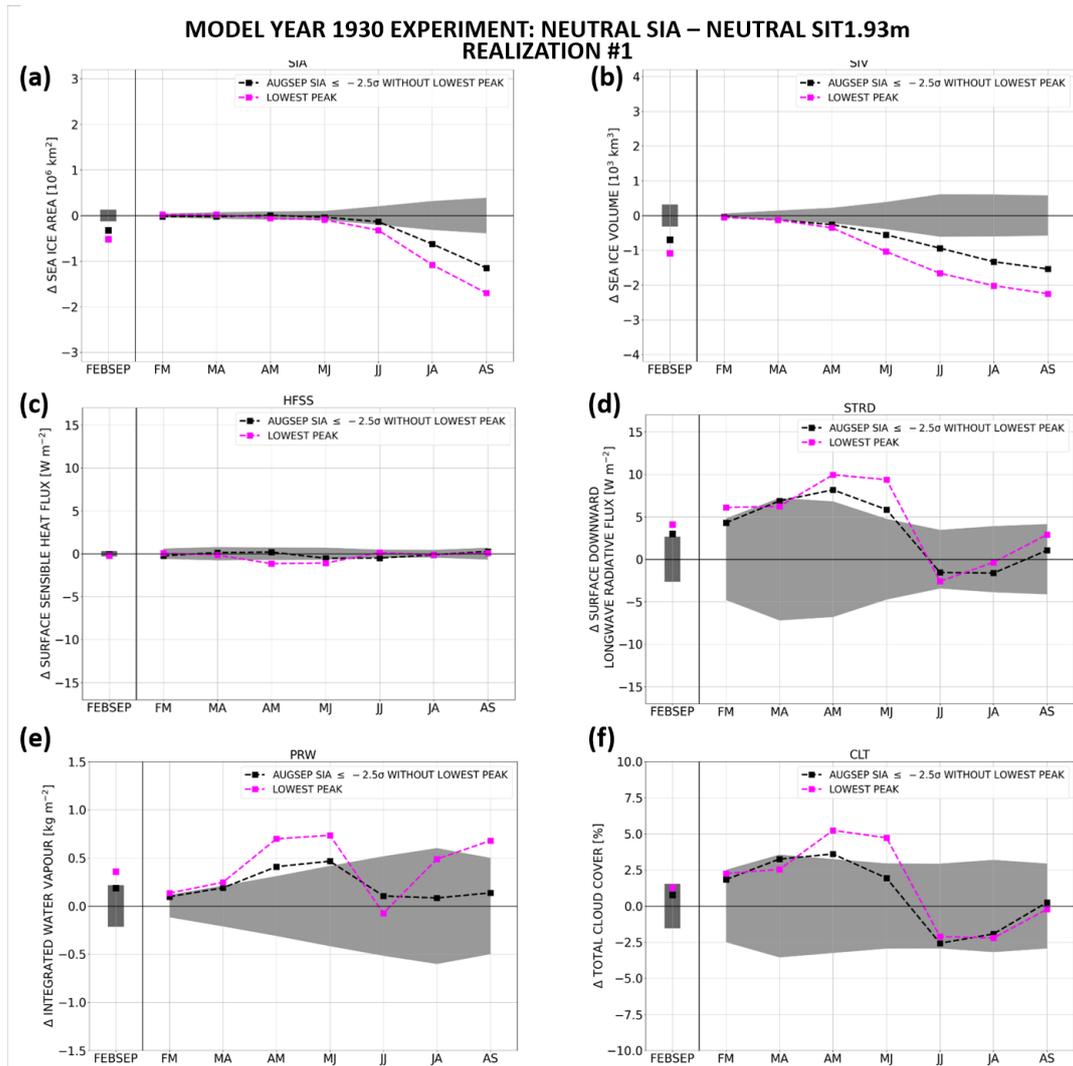
### S3: Bimodality in the rare event simulation



**Figure S4.** Rare event ensemble simulations initialized on 01 February 1930 of the control run. 15-day running mean of daily pan-Arctic sea ice area anomalies [ $10^6 \text{ km}^2$ ] relative to the control ensemble mean. The magenta lines show trajectories leading to the lower peak of the bimodal distribution of Figure 2(e) of the main manuscript and the dark black lines the trajectories leading to August-September mean sea ice area anomalies below  $-2.5$  control ensemble standard deviations excluding the lowest peak. The gray dashed line shows the 15-day running mean of the intra-ensemble standard deviation.



**Figure S5.** Ensemble simulations initialized on 01 February 1930 of the control run. August-September mean pan-Arctic sea ice area for (blue) the rare event simulation, (black) the control ensembles ( $N = 6000$  trajectories) and (red) the 3000-year control run for (a) rare event simulation realization #4 and (b) rare event simulation realization #7. The vertical lines show the mean of the distributions. The black and blue values indicate the smallest August-September mean sea ice area value in the control and rare event ensemble simulations respectively.



**Figure S6.** Rare event simulation realization #1 starting from a neutral winter initial condition. Composite mean anomalies of different quantities for all trajectories leading to an August-September mean sea ice area value belonging to (magenta) the "lower peak" of the rare event algorithm probability distribution function of Figure 2(e) and (black) for all trajectories leading to an August-September mean sea ice area anomaly equal or smaller than  $-2.5\sigma$  of the control ensemble excluding the magenta trajectories (see Figure S4 for the selected trajectories). The composites are computed using the estimator in Equation (3) of the main manuscript (see Sauer et al. (2024) for details about the computation of composites with the output of the algorithm). (a-b) Pan-Arctic sea ice (a) area and (b) volume. (c-f) Spatially averaged composite mean anomalies over all ocean grid boxes northern of  $70^\circ\text{N}$ . (c) Surface sensible heat flux and ( $[\text{W m}^{-2}]$ ; positive upwards) (d) downward longwave radiative flux anomalies [ $\text{W m}^{-2}$ ] (Direction-independent absolute values of the downward and upward fluxes are considered, i.e., a positive (negative) anomaly indicates a radiative flux that is stronger (weaker) in magnitude than the climatology). (e) Integrated water vapour [ $\text{kg m}^{-2}$ ] and (f) total cloud cover [%] anomalies. (a-f) Shading indicates the intra-ensemble standard deviation of the control ensemble and anomalies are computed with respect to the climatology of the control ensemble.

## 5 S4: Statistical significance

### Confidence intervals for probability estimates

We exploit  $M \in \{4, 5, 6, 7, 8, 9, 10\}$  rare event algorithm ensemble simulations to derive 95% confidence intervals around the estimate of a sea ice area anomaly amplitudes for a given probability level, i.e., we derive the confidence interval from  $M$  t-distributed sea ice area amplitude estimates available from the  $M$  experiments. We compute the confidence intervals as  $[\mathbb{E}_0(f) - t \frac{\sigma(f)}{\sqrt{M}}, \mathbb{E}_0(f) + t \frac{\sigma(f)}{\sqrt{M}}]$ , where  $\mathbb{E}_0(f)$  and  $\sigma(f)$  are the empirical average and standard deviation over  $M$  estimates and  $t$  takes the values  $t = 3.182$  for  $M = 4$ ,  $t = 2.776$  for  $M = 5$ ,  $t = 2.571$  for  $M = 6$ ,  $t = 2.447$  for  $M = 7$ ,  $t = 2.356$  for  $M = 8$ ,  $t = 2.306$  for  $M = 9$  and  $t = 2.262$  for  $M = 10$ . The variable sample size of amplitude-probability levels estimated with the rare event algorithm is due to the fact that the smallest probabilities obtained with a rare event simulation vary from one realization to the next due to sampling. Hence, the estimates of the probabilities and of their confidence intervals is based on the output of ten rare event algorithm experiments in a broad range of sea ice area values and on the output of four to nine experiments in the upper and lower amplitude range of sea ice area values. We only show estimates where at least four experiments are overlapping.

### Statistical significance of composite estimates

We use a two-sided t-test to estimate the statistical significance of the composite estimates presented in Figure 1(a) of the main manuscript. We use the 5% significance level. We subdivide the 3000-year control run Sauer et al. (2024) in  $M = 5$  ensembles with  $N = 600$  trajectories. We compute the t-value as  $t = \sqrt{M} \frac{\mathbb{E}_0(f)}{\sigma(f)}$ , where  $\mathbb{E}_0(f)$  is the empirical average of a quantity, i.e., the average over composite mean values obtained from the five ensembles, and  $\sigma(f)$  is the empirical standard deviation of that quantity, i.e., the standard deviation over composite mean values obtained from the five ensembles. We denote a composite mean value  $\mathbb{E}_0(f)$  as statistically significant at the 5% level if the absolute value of  $t$  is larger than 2.776.

## References

- 30 EUMETSAT Ocean and Sea Ice Satellite Application Facility: Monthly Mean Northern Hemisphere Sea Ice Area from EUMETSAT OSI SAF, v2p2, OSI-420, Norwegian Meteorological Institute, accessed 13 February 2024: [ftp://osisaf.met.no/prod\\_test/ice/index/v2p2/nh/osisaf\\_nh\\_sia\\_monthly.nc](ftp://osisaf.met.no/prod_test/ice/index/v2p2/nh/osisaf_nh_sia_monthly.nc), 2024.
- Sauer, J., Demayer, J., Zappa, G., Massonnet, F., and Ragone, F.: Extremes of summer Arctic sea ice reduction investigated with a rare event algorithm, *Climate Dynamics*, pp. 1–19, <https://doi.org/10.1007/s00382-024-07160-y>, 2024.