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100 kyr ice age cycles as a timescale-matching problem

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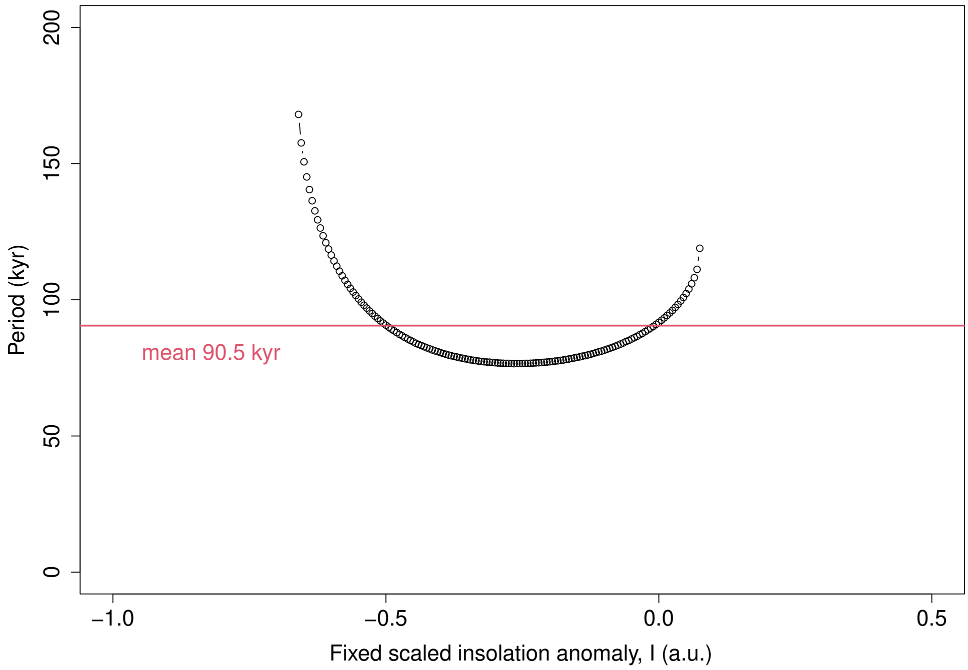

Fig. S1: Period of the self-sustained oscillations produced by the SO model for the fixed insolation forcing, $I(= \text{const.})$. The self-sustained oscillations occur for the range $-0.66 < I < 0.075$. The mean of the oscillation period over $-0.66 < I < 0.075$ is about 90.5 kyr, which is shown by the horizontal red line.

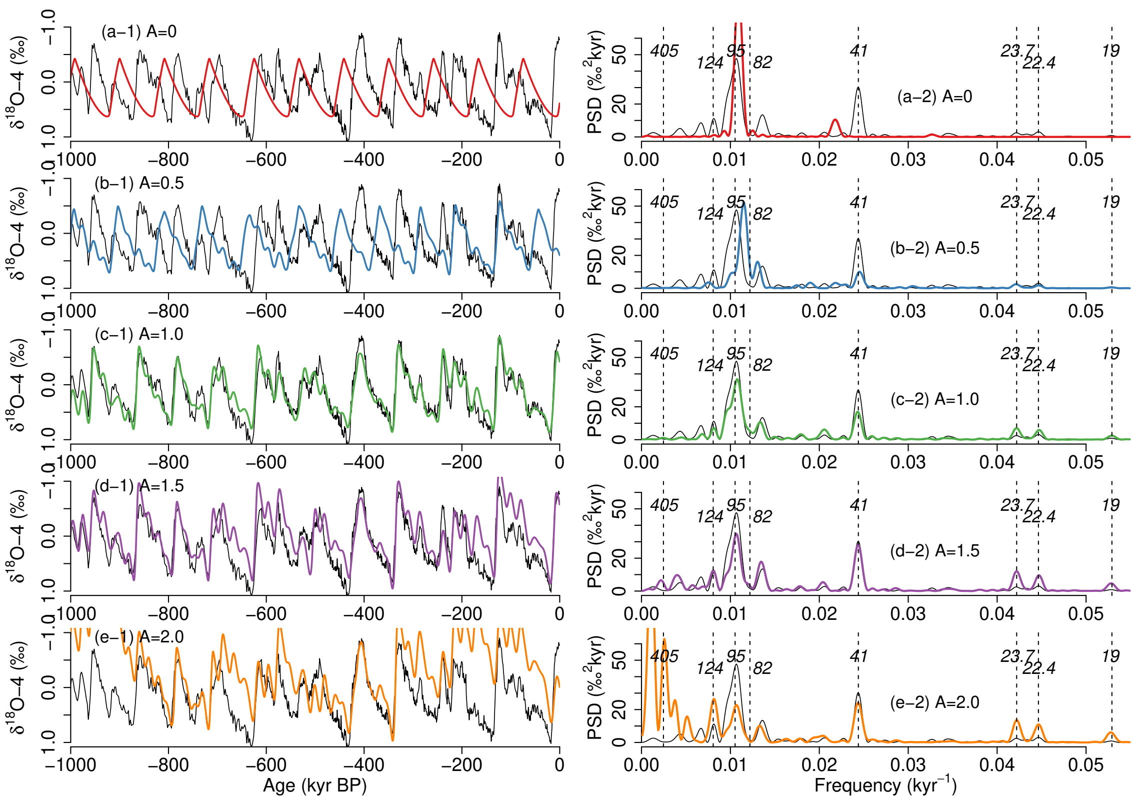

Fig. S2: SO model: The simulated δ^{18}O series are shown for different forcing amplitudes A on the left of (a) to (e). The black line is LR04 δ^{18}O record (Lisiecki and Raymo, 2005). The corresponding power spectral densities (PSD) are shown on the right. The vertical dashed lines and italic numbers show the positions of major astronomical frequencies and their periods.

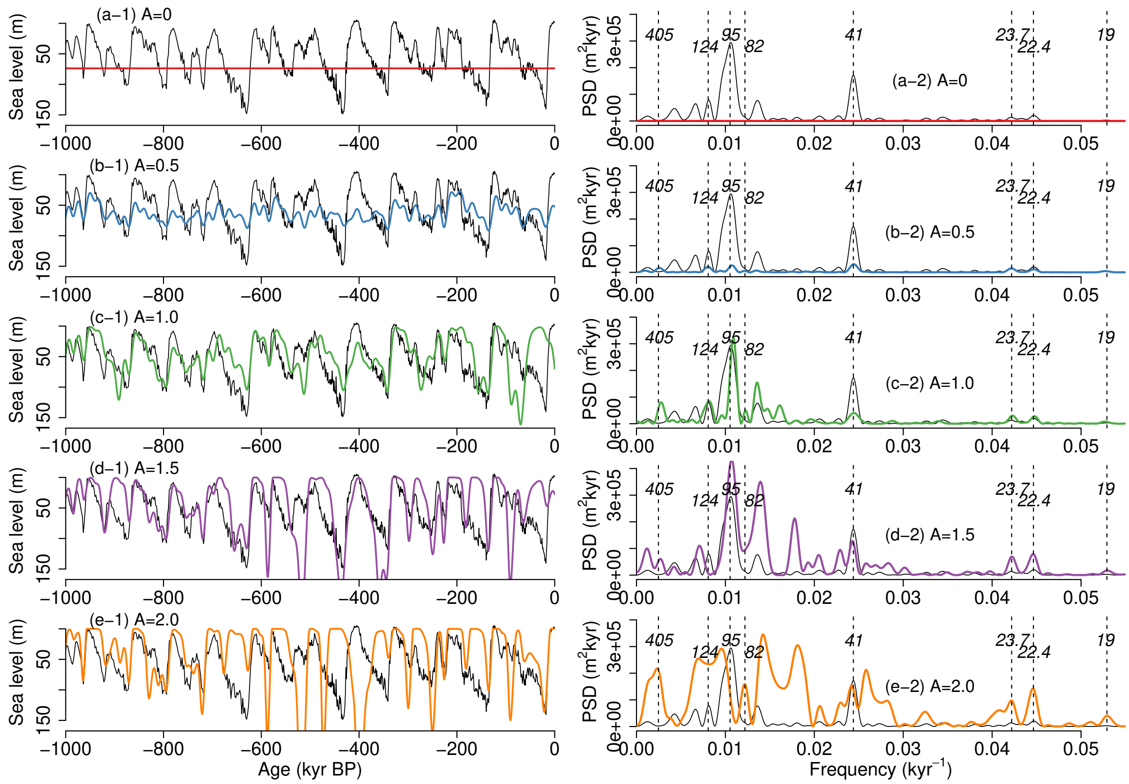

Fig. S3: VCV18 model: The simulated sea level are shown for different forcing amplitudes A on the left of (a) to (e). The black line is the sea level estimated from LR04 δ^{18}O record (Lisiecki and Raymo, 2005). The corresponding power spectral densities (PSD) are shown on the right. The vertical dashed lines and italic numbers show the positions of major astronomical frequencies and their periods.

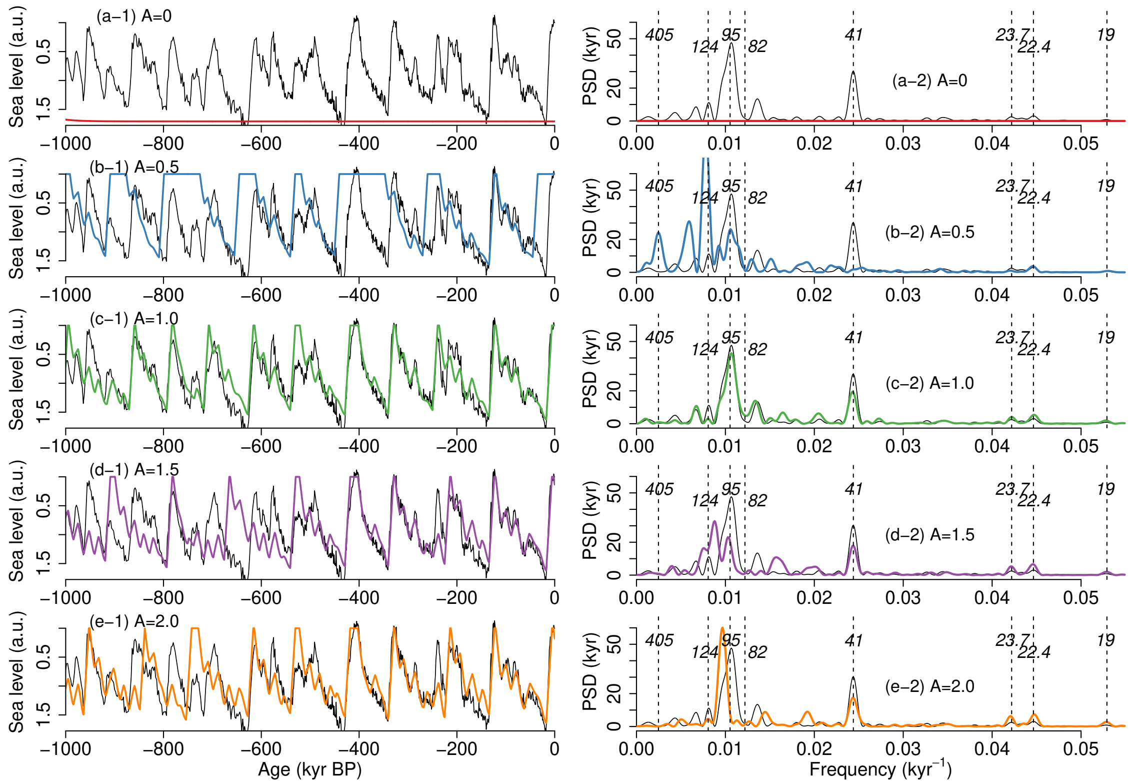

Fig. S4: G24-3 model (Ganopolski, 2024): The simulated sea level are shown for different forcing amplitudes A on the left of (a) to (e). The black line is the sea level estimated from LR04 δ^{18}O record (Lisiecki and Raymo, 2005). The corresponding power spectral densities (PSD) are shown on the right. The vertical dashed lines and italic numbers show the positions of major astronomical frequencies and their periods.

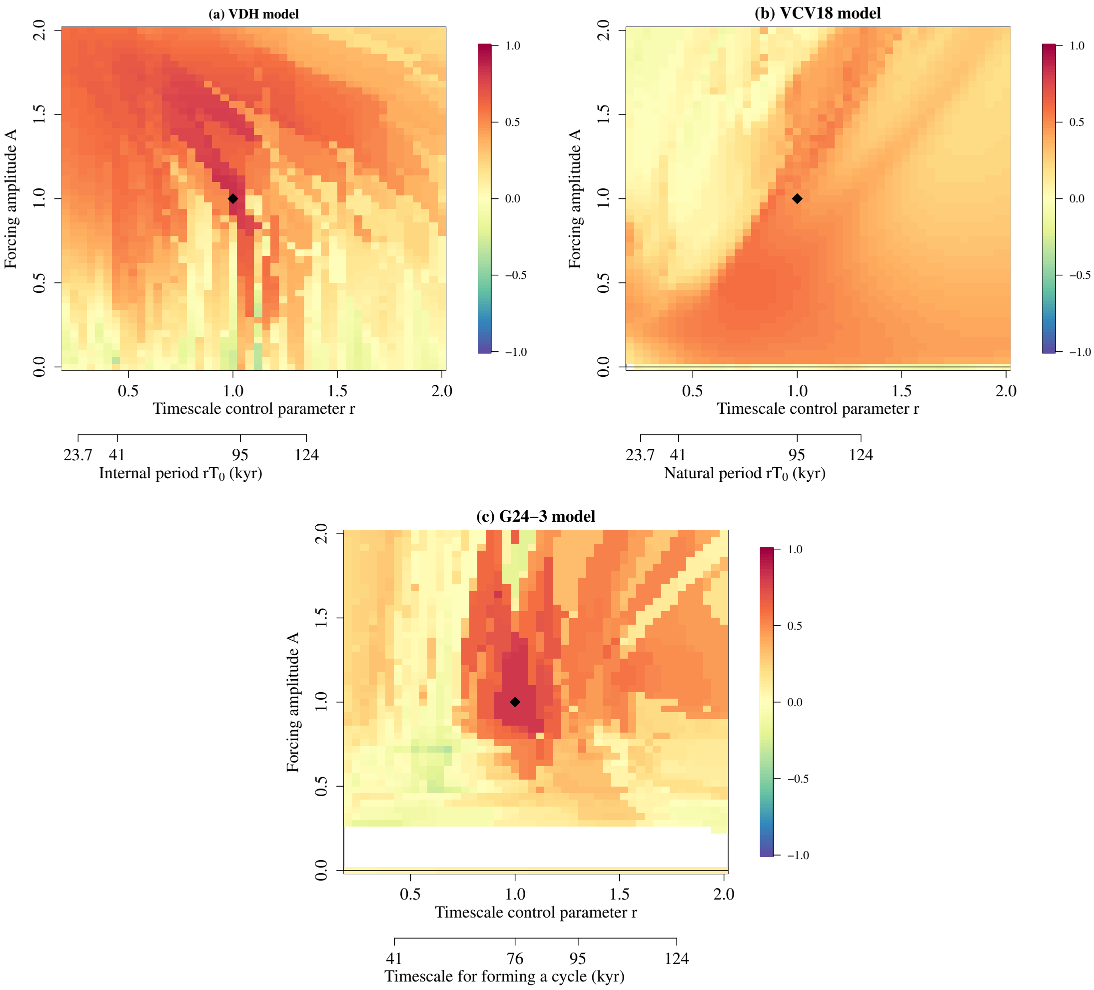

Fig. S5: Pearson's correlation coefficients between the simulations and the observed δ^{18}O series as a function of A and r. (a) SO model (synchronization mechanism). (b) VCV18 model (nonlinear resonance mechanism in a damped oscillatory system). (c) G24-3 model (nonlinear resonance mechanism in a bistable system with thresholds). The most realistic simulations are obtained at $A = 1$ and $r = 1$ (black diamond). A is the forcing amplitude. r is the parameter controlling the timescale of the underlying system. rT_0 is the scaled intrinsic timescale in the SO model and the VCV18 model. In the G24-3 model, the scaled timescale for forming a cycle is $T_{\mathrm{cyc}}(r) = 61.5r + 15$ kyr. In the white region in (c), the correlation coefficient cannot be determined because the simulated time series is constant.

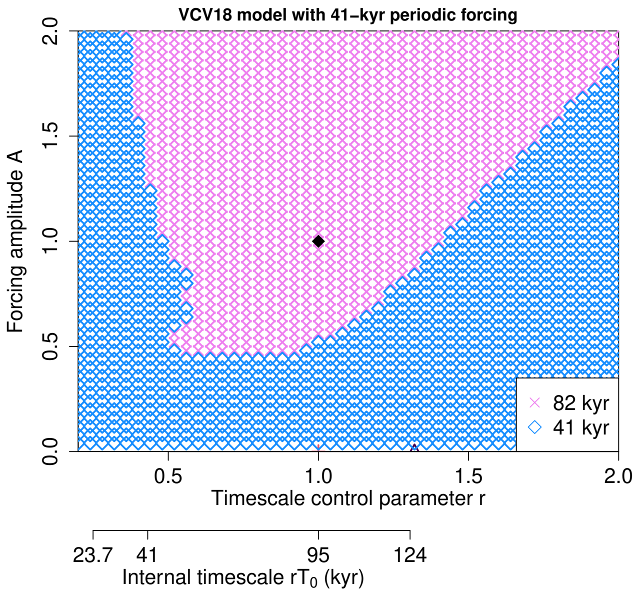

Fig. S6: Regime diagram in A–r space. The VCV18 model with the pure 41-kyr periodic forcing, $\sqrt{2}A\sin(2\pi t/41)$. The principal period of the simulated dynamics is shown by the symbols in the legend. The black diamond corresponds to $A = 1$ and $\tau = 1$. r is the parameter controlling the timescale of the underlying system. rT_0 is the natural periodicity of the scaled model.

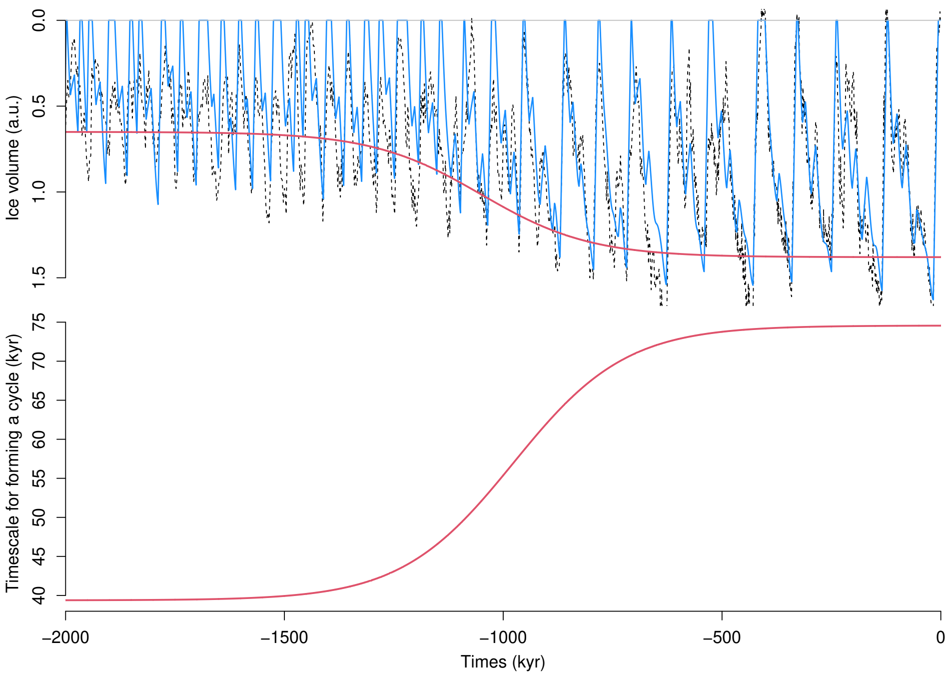

Fig. S7: Timescale of the G24-3 model (Ganopolski, 2024) through the last 2 Myr: (a) Simulated ice volume (blue) compared with the LR04 benthic δ^{18}O stack record (Lisiecki and Raymo, 2005). The critical ice volume v_c is shown by the red line. (b) Timescale for forming a cycle $T_{\text{cyc}} = t_1 \ln(V_g(0)/(V_g(0) - v_c)) + t_2 + 15$.

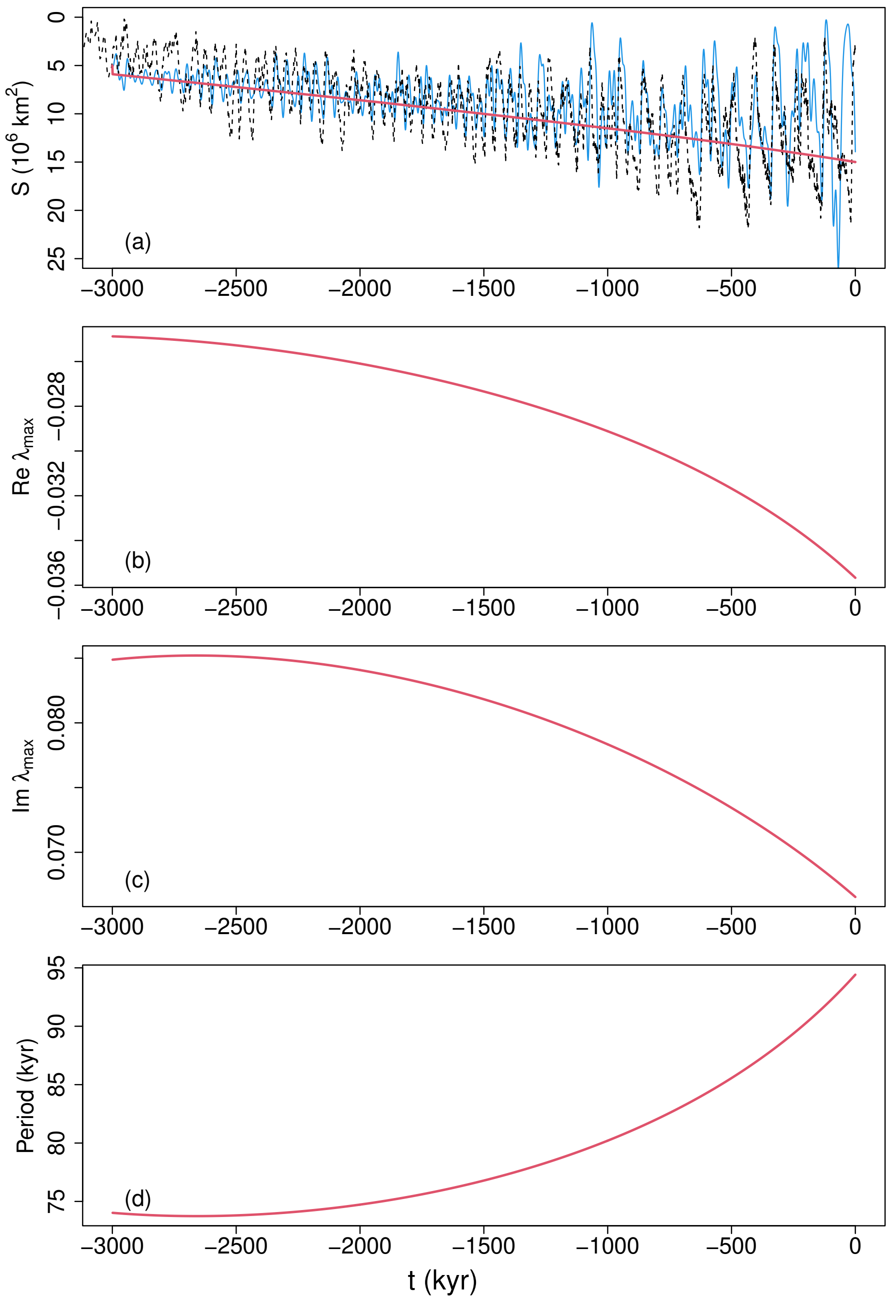

Fig. S8: Natural period of the VCV18 model (Verbitsky et al., 2018) through the last 3 Myr: (a) Simulated ice sheet area (blue) compared with the LR04 benthic δ^{18}O stack record (Lisiecki and Raymo, 2005). The stable state is shown by the red line. (b) Real part of the maximum eigenvalue, Re λ_{max}. (c) Imaginary part of the eigenvalue, Im λ_{max}. (d) Eigen period calculated as $2\pi/\mathrm{Im}\,\lambda_{\mathrm{max}}$.

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