



Supplement of

Carbon budget concept and its deviation through the pulse response lens

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S1 Scenario-dependent deviations - experimental setup

Optimization scheme - boundary conditions

The first boundary condition sets the total cumulative emissions at the year of optimization t^* to a fixed value F_{tot} , counting from the initial year t_0 , chosen as the year 2020 in RCP6.0. The condition on F_{tot} ensures that the deviation from the carbon budget stems only from the difference between the emission pathways, as it fixes the cumulative emissions to be equal at the end of both the minimization and the maximization run.

The second boundary condition provides the upper bound on the rate of change in emissions per year, effectively setting the allowed absolute slope of the emission pathway to be less than or equal to a prescribed value k . Hence, a trivial solution (e.g., emitting all of the emissions in one year) is avoided. The emission slope k is chosen such that its upper bound is 1 PgC/yr^2 , roughly corresponding to the emission reduction rate if the annual emissions were linearly reduced to zero between the years 2020 and 2030.

The combination of the restriction on k with the F_{tot} restriction will affect the run's feasibility. The higher the cumulative emissions and the lower the k is, the less feasible the run is. Moreover, the additional requirement that the emissions reach net-zero by t^* further negatively affects the feasibility. The feasibility limiting value of k will correspond to the run where both $T_{\text{max}}(t^*)$ and $T_{\text{min}}(t^*)$ are equal, as they come from the only possible and feasible scenario; hence, the scenario-dependent carbon deviation $T_d(t^*)$ is zero for that specific k . The higher k is, the more the range of possible pathway combinations increases, as does T_d .

The last boundary condition excludes negative emissions. This condition is utilized since Green's approach uses a pulse response generated under positive emissions. Nevertheless, for the sake of completeness, negative emissions will be allowed in the last part of the section to see how doing so affects the deviations.

Two settings of the scenario-dependent deviations

To examine the carbon budget interpretation, we distinguish between two additional sets of conditions that differ depending on how much we emit after the optimization year (t^*).

The first carbon budget interpretation is addressed as a net-zero budget case, which corresponds to the situation in which all of the carbon has been emitted up until the point in time of interest, and there are no other emissions afterward. This interpretation coincides more with a carbon budget as addressed by the IPCC, which indicates how much more carbon can be emitted while still reaching specific targets. In the corresponding emission scenario set, the emissions are bound to reach zero by the year t^* and stay zero from there onwards ($E(t \geq t^*) = 0$). Note, however, that this is not the case of calculating the ZEC deviations, even though the requirement is emission cessation. ZEC tells us what the temperature evolution will be following emission cessation. In the optimization program, however, one derives two maximally different possible temperatures in a specific year, stemming from different preceding emission choices, and the deviation comes from deducting the two. ZEC affects both boundary temperature cases equally, so when the two are subtracted to get the deviation $T_d(t^*)$, the effect of ZEC is also subtracted.

On the other hand, there is the transient budget case, in which only the momentary relationship between the current cumulative emissions and current temperature increase is of interest (as given by Eq. (1), for example). Therefore, in the optimization year t^* , emissions are free to take any value in transient budget case (within the limits of other constraints). The additional constraint on the emission pathway negatively affects the feasibility. Therefore, the transient budget case has more possible emission pathway combinations available compared to net zero, which means a higher expected T_d .

Deviation time evolution

The optimization procedure (Eq. (4)) calculates the extreme case of scenario-dependent deviations in one specific year t^* only. To see whether these deviations are persistent in time, an additional experiment is designed, one unique to the net-zero

approach. For unit of k specified in the setup above, the system is left to evolve for the next 50 years following the optimization year ($t^* = 2070$), without adding new emissions. Hence, $T_d(k)$ is allowed to evolve freely in time, while keeping cumulative emissions at the same level. In this way, one can see how the scenario-dependent deviation obtained in t^* changes in time.

45 **Run configuration**

50 Preceding the initialization of the optimization program, the FaIR model was historically forced from the preindustrial period (the year 1850) until 2020 under the RCP6.0 emission scenario. The quasi-historical run is dynamically separated from the optimization run since, in the former, emissions are prescribed, not generated by the program. The two runs coincide in the year 2020, where the values of the historical run's variables are translated into the initial conditions of the variables of the FaIR's optimization run. Hence, $t_0 = 2020$ in Eq. (4) and the initial emissions value of the optimizer run equals to $E_0 = E_{\text{RCP6.0}}(2020)$. The initial temperature at t_0 is $T_0 = 0.96$ K, with the associated cumulative emissions counting $F_0 = 584$ PgC.

S2 Optimization year sensitivity

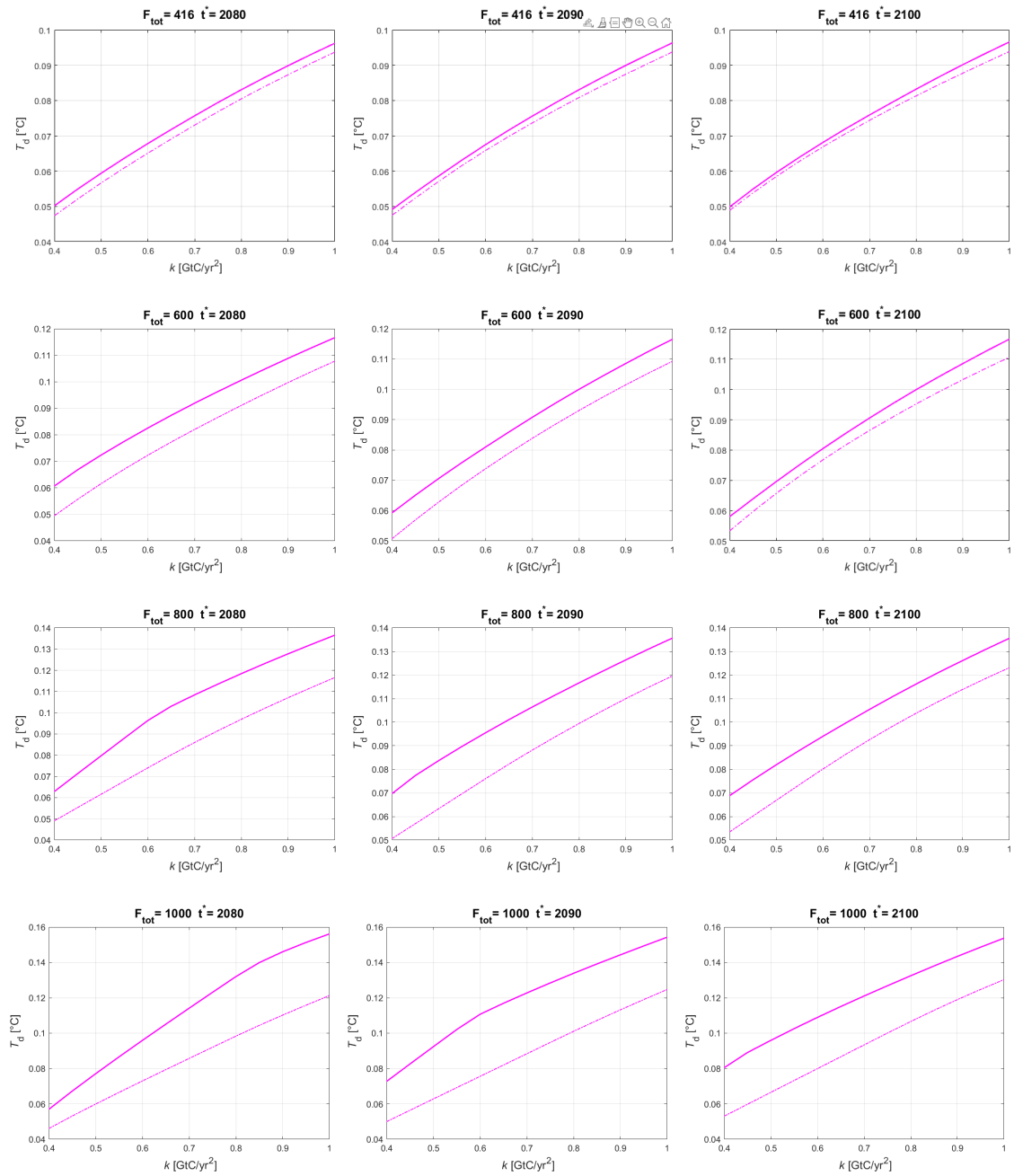


Figure S1. Maximal scenario dependent deviations for different optimization years and total cumulative emission choices, under transient budget case. One can detect that the optimization year choice does not affect the generated deviations, except for feasibility limit that becomes more prominent the lesser t^* , or prominently, the higher F_{tot} is.