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Supplement of

Stratospheric ozone and quasi-biennial oscillation (QBO) interaction with the tropical troposphere on intraseasonal and interannual timescales: a normal-mode perspective

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Here we provide an example of PDC analysis for a synthetically generated time series.

PDC analysis of a synthetic time series

In order to exemplify the use of partial directional coherence (PDC) method we present an analysis of a times series generated by iterated functions in such a way that we know beforehand the direction of the causality.

Consider a bivariate time series $(x_1(n), y(n)) \in \mathbb{R}^2$ for all times , generated iteratively:

$$x_1(n) = 1.601x_1(n-1) - 0.98x_1(n-2) + W_1(n)$$
$$x_2(n) = 0.85\, x_2(n-1) - 0{,}05x_1(n-1) + W_2(n)$$

(S1)

Where W_1 and W_2 are uncorrelated Gaussian white noise with distribution $\mathcal{N}(0,1)$. From equation (S1) we see that the present (time n) of the x componet depend only on the past component of the x component itself at time $(n-1)$ and $(n-2)$. For the y component the present state depends on the past of y at lag one ($x_2(n-1)$) and of the past of the component x at lag one ($x_1(n-1)$). Therefore for this time series x_1 causes $x_2$but x_2 doesn't cause x_1.

The generated time series were simulated with 1024 points. A VAR(2) model (bivariate autoregressive model with two lags) was fitted to the data using the Hannan-Quinn criterion. The fitted model passed Portmanteau test with 5% confidence level.

In figure (S1) we present the PDC analysis of the time series generated by formula (S1). A detailed explanation on how to read a PDC plot is given in the caption of the figure.

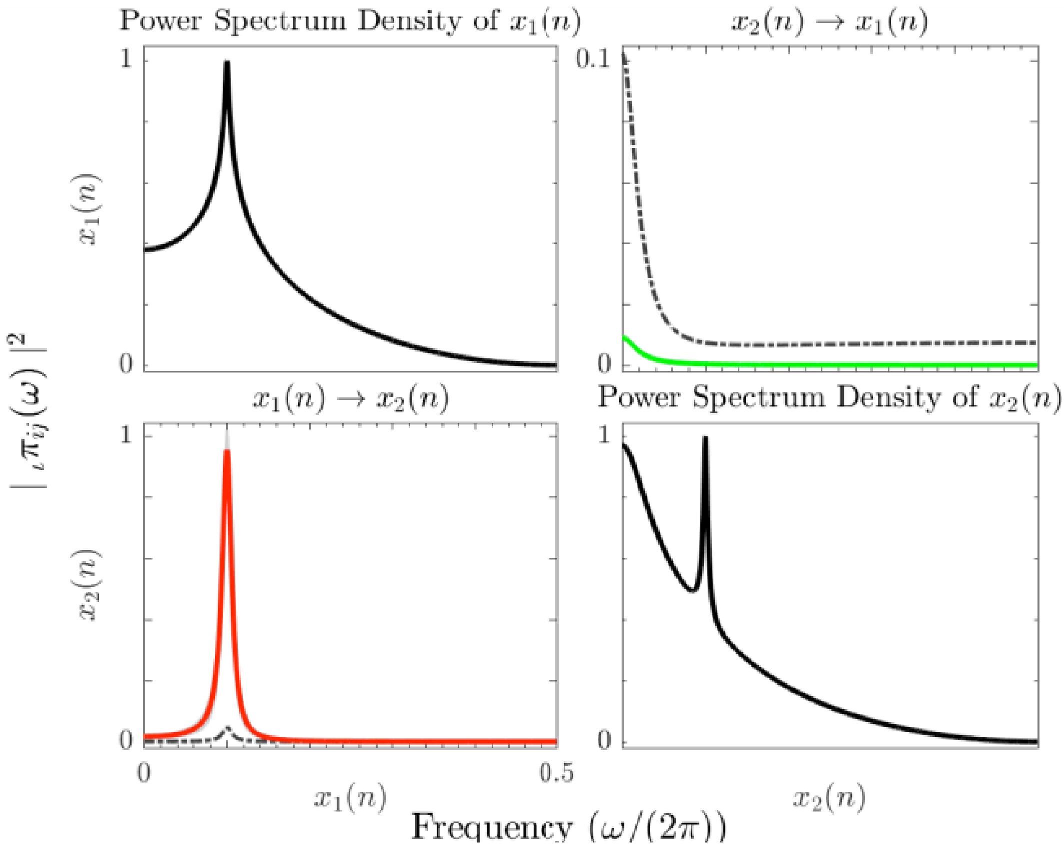

Figure S1: PDC analysis of a synthetically generated time series through equation (S1). The horizontal axis is given in frequency (non dimensional) and the vertical axis represents the iPDC coefficient. ${}_\iota\pi_{11}(\omega)$ represents the power spectral density of the variable x_1 and is presented at the top to the left of the panel, respectively ${}_\iota\pi_{22}(\omega)$ represents the power spectral density of the variable x_2 and is presented at the bottom to the right of the panel. Coefficient ${}_\iota\pi_{12}(\omega)$ represents the influence of the variable x_2 on x_1 presented at the top to the right of the graph. Coefficient ${}_\iota\pi_{21}(\omega)$ represents the influence of the variable x_1 on x_2 presented at the bottom to the left of the graph. The value of the PDC at a given frequency ω is given by the red(green) curve. If the curve is red at a given frequency it means that there is causality from one variable to the other at that frequency within the chosen confidence level, conversely if the curve is green at a given frequency it means there is no causality from one variable to the other at that frequency. The dashed curve represent the statistical threshold above which the causality becomes statistically significant, in other words, if the PDC curve is above the dashed curve at a given frequency the causality is significant and the PDC curve is red, respectively if the PDC curve is under the dashed curve causality is not statistically significant and the curve is green.

From the graph on figure (S1) we conclude that variable x_1 influences the variable x_2 at the whole spectrum since the PDC curve (at the bottom to the left) is red at all frequencies using 5% of confidence level. On the other hand x_2 doesn't influence x_1 because PDC curve (at the top to the right) is green at all frequencies. This agrees with what we already knew from the definition of the time series from equation (S1).