

Supplement of Earth Syst. Dynam., 11, 525–535, 2020
<https://doi.org/10.5194/esd-11-525-2020-supplement>
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Supplement of

On the interconnections among major climate modes and their common driving factors

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Supplementary Note 1

Here we use an ideal model to show that varying the embedding dimension can serve as a sensitivity analysis of the robustness of the significant peak-periods of SFA-derived signals. This part is meant to support our analysis and conclusions drawn in the main text (**Figs. 3 and 4**)

We consider a logistic map with two-level structures that contains two time-varying driving factors $a(t)$ and $b(t)$:

$$x(t + 1) = a(t)x(t)(b(t) - x(t)), (t = 1, 2, \dots, n) \quad \dots(1)$$

where

$$a(t) = 3.5 + 0.4\cos(2\pi t/78) \quad \dots(2)$$

$$b(t) = 0.998 + 0.024\cos(2\pi^2 t/78) \quad \dots(3)$$

From formulas (2) and (3), we compute the true periods of $a(t)$ and $b(t)$ to be about 78.00 and 24.84 steps, respectively. First, we set the embedding dimension m to 6 for the SFA and extract the driving-force signal from $x(t)$, which is denoted as sfa- $x(t)$.

Figs. S1 (a) and S1 (b) show the time series of the parameters ($a(t)$ and $b(t)$) in the Logistic map with two-level structure. **Fig. S1 (c)** shows the non-stationary time series $x(t)$, which is controlled by $a(t)$ and $b(t)$. The driving signal of $x(t)$ when setting embedding dimension $m=6$ (i.e. sfa- $x(t)$) is shown in **Fig. S1 (d)**. From **Fig. S1**, we can see that the temporal evolution of sfa- $x(t)$ is neither $a(t)$ or $b(t)$, but the combination form of them. As mentioned in the main text, only one embedding dimension is not enough to demonstrate the effectiveness of SFA. To confirm the results derived from SFA and wavelet analysis, we carry out a

sensitivity test to identify the significant peak-periods of SFA-derived signals by setting the embedding dimensions from 2 to 16. The peak-periods in the time-averaged power spectrum of SFA signals (significant at a 0.05 significance level) with different embedding dimensions are shown in **Table S1**. The results show that with different embedding dimensions, SFA signal is either the combination of true independent driving forces or the slowest driving force factor only (this is due to the fact that higher embedding dimensions tend to smooth out the high-frequency oscillations). Thus, based on the above ideal model, we demonstrate the effectiveness and robustness of the approach that combines SFA with wavelet analysis: the sensitivity analysis with varying embedding dimensions can provide robust results. While further studies are needed to quantify the uncertainties of the peak-periods, it appears that the derived in this work peak-periods are rather robust.

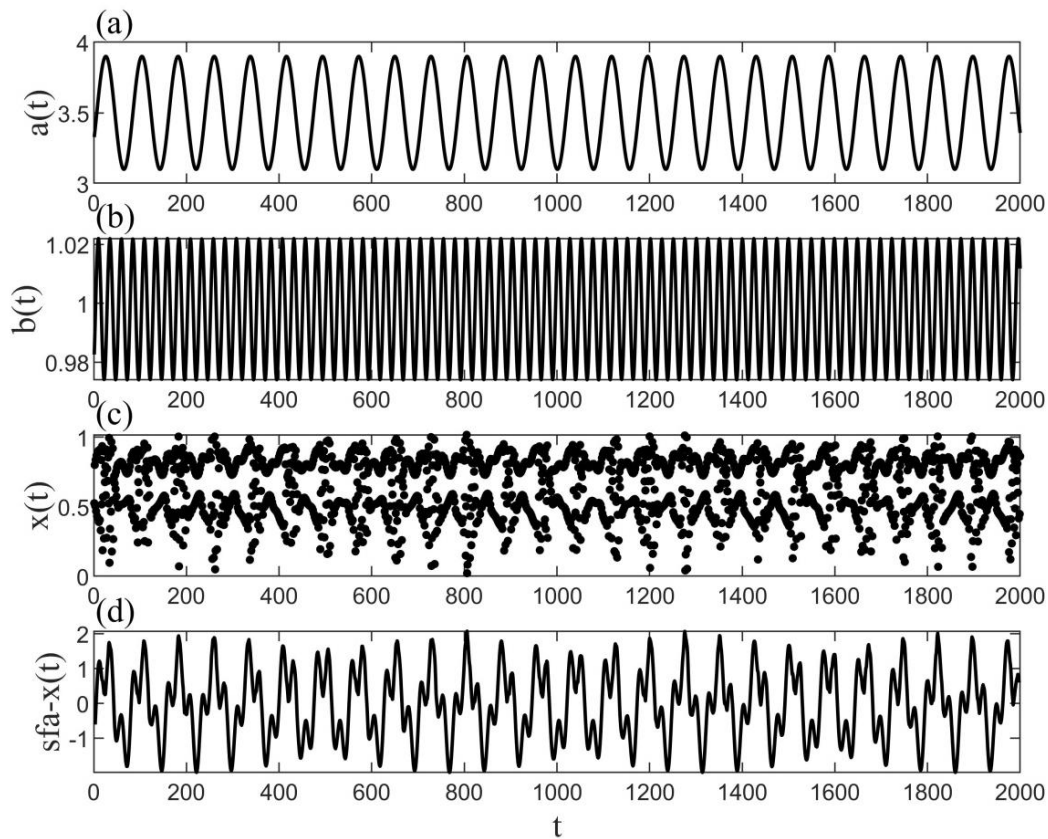


Figure S1: The time series of the parameters in the Logistic map with two-level structure. (a) Time series of true driving forces $a(t)$; (b) Time series of true driving force $b(t)$; (c) The non-stationary time series $x(t)$, which is controlled by $a(t)$ and $b(t)$; (d) The slow feature $sfa-x(t)$ derived from $x(t)$ by using SFA ($m = 6$ and $t = 1$).

Table S1: The peak-periods in the time-averaged power spectrum of SFA signals (statistically significant at a 0.05 significance level) when setting different embedding dimensions (unit: step).

m	Periods	
	(reserve two decimal fractions)	
2	23.38	78.62
3	23.38	78.62
4	23.38	78.62
5	23.38	78.62
6	23.38	78.62
7	23.38	78.62
8	23.38	78.62
9	23.38	78.62
10	78.62	
11	78.62	
12	78.62	
13	78.62	
14	78.62	
15	78.62	
16	78.62	

Supplementary Note 2

The above illustration with the logistic equation we believe is very instructive. Nevertheless, we present some additional results and arguments that demonstrate that it is also a representative example, and not a way to obtain an easy answer using a simple system that may not be as easy with a more complex system.

Fig. S2 shows the time-averaged power spectrum of each parameter in **Fig. S1** (i.e. of the actual time series).

We can see that the significant peak periods in **Fig. S2(a)** and **Fig. S2(b)** are very close to the true period of $a(t)$ and $b(t)$. **Fig. S2(c)** is the power spectrum of $x(t)$ and **Fig. S2(d)** is the power spectrum of $sfa-x(t)$. The significant peak-periods of $sfa-x(t)$ are equal to the ones of $a(t)$ and $b(t)$. It means that the significant peak-periods of SFA-derived signals well represent the periodic characteristics of detected driving forces. **Fig. S3** is the same as **Fig. 2** but for the raw time series of the six indices. Note that this **Fig. S3** should not be confused with **Fig. 2** of the main text, which is the average power spectra of the SFA extracted signals. Its purpose is to show that the logistic example is as complex as the indices and maybe even more. As we can see in **Fig. S2(c)** the spectra of $x(t)$ are more “white” than those of the indices in **Fig. S3**.

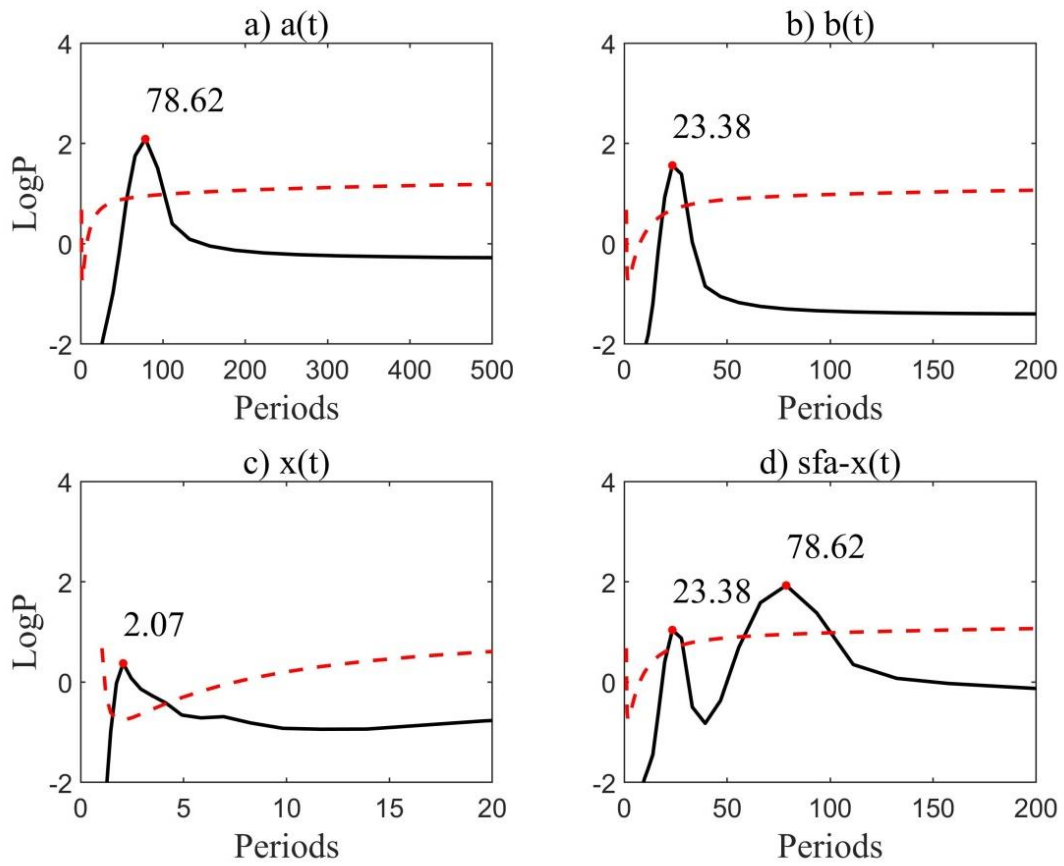


Figure S2: From (a) to (d) are the time-averaged power spectra (black lines) of $a(t)$, $b(t)$, $x(t)$ and $sfa-x(t)$, respectively. The red dashed lines show the significance test at a 0.05 significance level for wavelet analysis. The red dots indicate the peak-periods that pass the significance test. The values of corresponding period are denoted along with red dots.

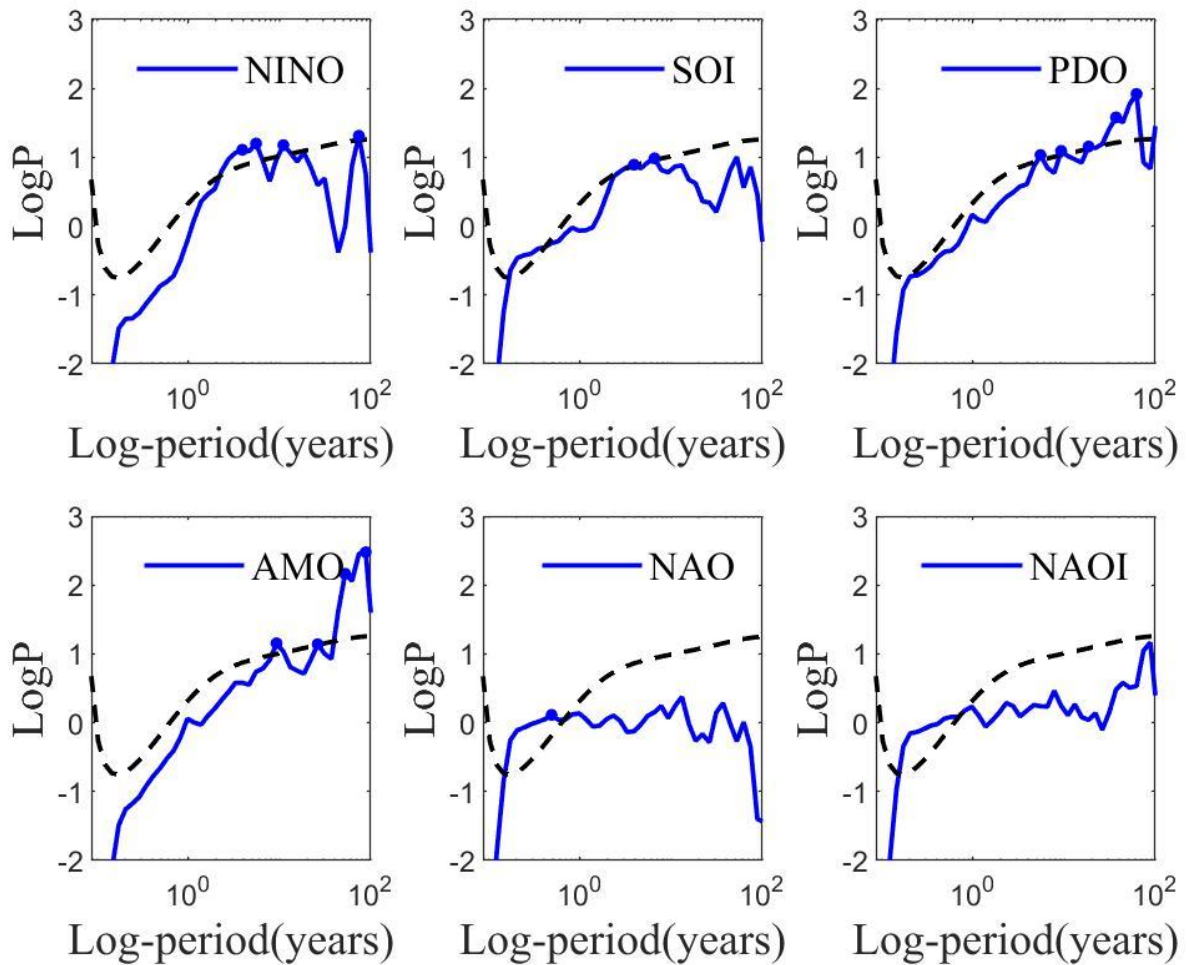


Figure S3: The time-averaged power spectrum of the raw time series of the six climate indices. Significant periodicities (blue dots) at a 0.05 significance level (black dashed lines) are also indicated.

Supplementary Note 3

As a final note, we would like to address an interactive comment by Prof. Dmitry Sonechkin, which is part of the discussion package. In that discussion, it is suggested that the Chandler wobble period and the Luni-Solar nutation, may be behind QBO and ENSO, and thus they may be the prime causes of the macroscale climatic processes instead of QBO and ENSO (Serykh and Sonechkin, 2019; Serykh et al., 2019).

As we mention in our paper, the issue of the interaction of climate modes and climate variability is lately becoming extremely important, and more and more insights will be forthcoming. We believe that Prof. Sonechkin's group research is important and should be taken into consideration in the grand scheme of climate modes and climate variability.

References

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